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Decentralized Suboptimal Approach to the Control of Large- Scale Systems

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ABSTRACT. The finite moving horizon (MH) is proposed for the suboptimal decentralized linear quadratic problems. The technique of MH is based on a finite stage solution of the Riccati difference equation. Numerical results show that MH technique has fast computability properties. The proposed technique is tested with a numerical example.

1. Introduction

Optimal control theory has been employed for suboptimal decentralized control of multivariable systems. In the case of linear quadratic gaussian designs, one has to have reliable algorithms that solve very high-dimensional Riccati equations [1, 2]. The major difficulty is the computational requirements of solving the Riccati equation at each time interval [3]. To overcome this difficulty many methods [4 - 10] have been proposed to obtain nearly optimal decentralized control for the high order systems. In [11], the Riccati equation is iterated once at each time step, which reduces the computational load of solving the steady-state Riccati equation. However, the quality of control and energy conservation at each time step may not be optimal and good transient behavior is not always assured. It would be interesting to simplify this calculation by the use of MH optimization schemes which often result in both robustness and fast computability [12, 13]. The idea of MH method was firstr proposed by Kleinman [14], and was developed by others [15 – 18]. In this paper, the suboptimal control law for a large scale system is derived from the suboptimal law of the subsystems obtained by solving Riccati equations which make use of MH.

The behavior of the algorithm is illustrated by a numerical example. The properties of the suboptimal decentralized controller are studied using a large-scale system consisting of N subsystems associated with the MH technique.

2. Regulation

Consider a large-scale system consisting of N subsystems described by :

$$X_i\,(k+1) = A_{ii}\,X_i\,(k) + B_i\,U_i\,(k) + \sum_{\substack{J=1 \\ J\neq i}}^{N} A_{ij}\,X_j\,(k)$$

$$y_i\,(k) = C_i\,X_i\,(k), \qquad i = 1,2,\ldots\ldots N \tag{1}$$

where $X_i\,(k) \in R^{ni}$, $U_i\,(k) \in R^{mi}$ and $y_i\,(k) \in R^{ri}$, C_i have full row rank.

The MH controller is defined as the one which minimizes the quadratic loss function as follows by [3]:

$$J_i = X_i^T\,(k+N)\,F_i\,X_i\,(k+N) + \sum_{i=N}^{K+N-1} X_i^T\,(k)\,Q_i\,X_i\,(k) + U_i^T\,(k)\,R_i\,U_i\,(k) \tag{2}$$

where $F_i^T = F_i \rangle 0$ is a positive definite matrix called the " final state penalization matrix" , $Q_i^T = Q_i \rangle 0$ is a positive semidefinite matrix and $R_i^T = R_i \rangle 0$ is a positive definite matrix. All the matrices subject to system equation (1). The control law in the case of state feedback is given by [1]:

$$U_i^m\,(k) = -\,(R_i + B_i^T\,P_{i(N-1)}\,B_i)^{-1}\,B_i^T\,P_{i\,(N-1)}\,A_i\,X_i\,(K) \tag{3}$$

where $P_{i(N-1)}$ is the stage solution, hence

$$K_i = -\,(R_i + B_i^T\,P_{i(N-1)}\,B_i)^{-1}\,B_i^T\,P_{i\,(N-1)}\,A_i \tag{4}$$

$$\begin{aligned} P_i^{J+1} &= A_i^T\,P_i^J\,A_i - A_i^T\,P_i^J\,B_i\,[(R_i + B_i^T)\,P_i^j\,B_i\,]^{-1}\,B_i^T\,P_i^J\,A_i + Q_i \\ &= \psi_i\,(P_i^J) \end{aligned} \tag{5}$$

starting with $P_i\,(0) = F_i\,.\,N_i$, where N_i is an any positive integer . For the existence of a stabilizing MH control , it is required that

$$F_i \geq \psi_i\,(F_i) \tag{6}$$

which guarantee the ordering :

$$P_i^j \geq P_i^{j+1} \qquad (7)$$

so that the MH is stable.

3. Computational Load

The solution of the steady-state Riccati equation:

$$P = Q + A^T P A - A^T P B (R + B^T P B)^{-1} - B^T P A \qquad (8)$$

Using the matrix sign algorithm, it can be summarized as follows :

$$I = \begin{bmatrix} A^{-1} & A^{-1}BR^{-1}B^T \\ QA^{-1} & A^T + Q^{-1}BR^{-1}B^T \end{bmatrix} \qquad (9)$$

The matrix sign algorithm is :

$$\text{Sign} (\Gamma) = \text{Lim} (\Gamma^j - I_{2n}) (\Gamma^j + I_{2n})^{-1} \equiv \Gamma_j \qquad (10)$$

define a matrix W given by :

$$W_J = \Gamma_J + I = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \qquad (11)$$

where I = block diagonal $[I_n, - I_n]$.

The approximate Riccati gain matrix P_j is given by :

$$P_j = (W_{21}) (W_{11})^{-1} \qquad (12)$$

In the moving horizon method the corresponding gain matrix P(N) is given by :

$$P(N) = \sum_{i=0}^{N-1} (A^{-1})^i (BR^{-1}B^T)^i (A^T)^{-i} \qquad (13)$$

At each iteration the matrix sign algorithm requires one matrix inversion of order 2nx2n, plus two matrix inversions of order nxn, to solve equation (8) and (9), while the moving horizon method requires only one matrix inversion of order nxn. Taking into consideration that the matrix inversion calculation is proportional to n^2, it becomes clear that the moving horizon method is about seven times faster.

4. Algorithm

1- Choose an initial set of stabilizing decentralized state feedback gains K_i (0)
2- Calculate P_i^{J+1} , $i=1,2,\ldots\ldots N$,
J = number of iteration from equation (4)
3- Calculate the K_i state feedback gains at P_i^{J+1}
4- Test for $P_i^{J+1} = \psi_i (P_i^J)$ from equation (5).
stop the program otherwise go to step 2 until $P_i^{J+1} = P_i^J$
i.e converges to the steady state to obtain the suboptimal decentralized state feedback gains.

Example

Consider a high order system consisting of three subsystem defined by the matrices as follow as in [19]:

$$A_{12} = \begin{bmatrix} 1.010 & 0.0 & -0.042 & -0.009 \\ -0.079 & 1.0 & 0.556 & 0.138 \\ 0.027 & 0.0 & 2.020 & 0.501 \\ 0.112 & 0.0 & 4.090 & 2.020 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0.042 & 0.132 \\ -0.411 & -0.072 \\ 0.400 & 0.312 \\ -1.045 & -0.076 \end{bmatrix}$$

$$A_{13} = \begin{bmatrix} 1.040 & 0.0 & -0.127 & -0.029 \\ 0.232 & 1.380 & 0.340 & 1.0 \\ 0.116 & 0.0 & 1.220 & 0.502 \\ 0.473 & 0.0 & 0.850 & 1.230 \end{bmatrix}, \; C_1 = I$$

$$A_{21} = \begin{bmatrix} 1.110 & 0.0 & 0.064 & 0.016 \\ -0.581 & 1.0 & -0.330 & -0.085 \\ 0.115 & 0.0 & 1.220 & 0.502 \\ 0.486 & 0.0 & 0.893 & 1.230 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0.581 & 0.402 \\ 0.740 & 0.604 \\ 0.405 & 0.315 \\ 0.865 & 0.014 \end{bmatrix}$$

$$A_{22} = \begin{bmatrix} 0.875 & 0.301 & 2.020 & 0.182 \\ -0.526 & 0.267 & 2.320 & 1.130 \\ -0.387 & 0.0 & 0.332 & 0.502 \\ -1.620 & -0.387 & -3.410 & 0.299 \end{bmatrix}, C_2 = I$$

$$A_{23} = \begin{bmatrix} 1.350 & 0.0 & 0.265 & 0.001 \\ -0.921 & 1.0 & 0.638 & -0.019 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.064 & 0.0 & 1.390 & 0.879 \end{bmatrix}$$

$$A_{31} = \begin{bmatrix} 0.999 & 0.0 & 0.041 & 0.0 \\ -0.921 & 1.0 & 0.638 & -0.019 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ -0.576 & 0.0 & 0.187 & 1.010 \end{bmatrix},$$

$$A_{33} = \begin{bmatrix} 0.228 & 0.725 & 0.298 & -0.448 & \\ -2.940 & 1.820 & 6.730 & -0.495 & 0.298 \\ -0.425 & 0.0 & 0.763 & 0.504 & \\ -1.610 & -0.425 & -0.549 & 0.730 & \end{bmatrix}, B_3 = \begin{bmatrix} 0.260 & 0.242 \\ 0.156 & 0.486 \\ 0.405 & 0.315 \\ -0.550 & 0.173 \end{bmatrix}, C_3 = I$$

Let $Q = 0.045\ I_{4X4}$, $R = 101\ I_{2X2}$ and the horizon N is chosen to be N=5. The suboptimal decentralized feedback of this example is carried out through the use of computer program devised for the present algorithm. The results of the computation method converged to the suboptimizeing feedback gains as follows:

$$F1 = \begin{bmatrix} -0.000022 & 0.000044 & -0.00131 & -0.000057 \\ -0.000022 & 0.000010 & 0.00390 & -0.000240 \end{bmatrix}$$

$$F2 = \begin{bmatrix} -0.000580 & -0.000510 & -1.25700 & -0.270000 \\ -0.000390 & -0.000373 & 0.00390 & -0.366100 \end{bmatrix}$$

$$F3 = \begin{bmatrix} -3.190000 & -0.436600 & -0.00159 & 0.000450 \\ 1.906000 & -1.659000 & -0.00042 & -0.000786 \end{bmatrix}$$

From the curves, it is clear that the states of the variables (VAR_1 ,VAR_2, VAR_3 and VAR_4) for the suboptimal closed- loop (A+BFC) $subsystem_1$, $subsystem_2$, and $subsystem_3$ reach steady state in a short time.

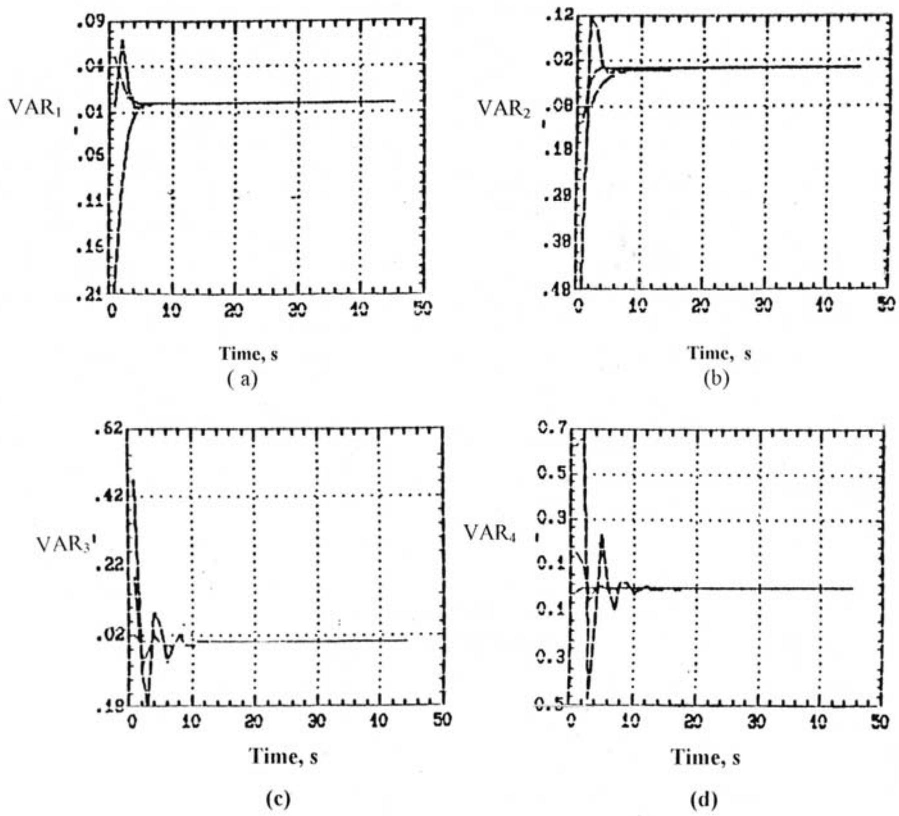

Fig. 1. Response of (a) VAR_1, (b) VAR_2, (C) VAR_3 and (d) VAR_4 for the suboptimal closed-loop three subsystems.

Conclusions

A finite moving horizon technique for the suboptimal state feedback is applied to large scale linear systems to obtain a solution for the decentralized control problem. The suboptimal control law for the large scale system is obtained from the suboptimal laws of the subsystems. This method is suboptimal, very fast and easy to implement with low computational requirements. A simulated example is given in which the procedure is applied to suboptimal decentralized control.

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الوصول لحاكم لا مركزي شبه مثالي للأنظمة الكبيرة المقياس

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المستخلص : تتكون معظم نظم التحكم الآلي الحديثة من عدة أنظمة مرتبطة بعضها ببعض لتكون نظام تحكمى كبير مثل أنظمة الاتصالات والشبكات والقوى الكهربائية . وللتحكم في مثل هذه الأنظمة الكبيرة بالطريقة العادية ، تستخدم عمليات حسابية معقدة وطويلة بالإضافة إلى ارتفاع التكلفة ، وللتغلب على هذه المشكلات تم استخدام الطريقة اللامركزية ، أي عمل أنظمة فرعية لتبسيط مثل هذه النظم الكبيرة . ولذلك كان الغرض من هذا البحث هو تصميم حاكم لا مركزي شبه مثالي للأنظمة الكبيرة المقياس . ينبني هذا الحاكم على أسلوب المستوى المتحرك المحدد بهدف الحصول على أنسب تكلفة مثلى للنظام . تم استخدام نظام ذو نطاق واسع يتألف من عدد من الأنظمة الفرعية . تم إحراز قاعدة حاكمة شبه مثالية للنظام الواسع ككل من القواعد شبه المثالية للأنظمة الفرعية . وقد جرى إعداد برنامج محاكاة على الحاسوب مع استنتاج العلاقات الرياضية المختلفة للطريقة المقترحة . وقد امتدت الدراسة كذلك بالتطبيق على مثال من أنظمة التحكم لاختبار العمليات الحسابية المستخدمة . وأثبتت النتائج التي تم التوصل إليها سهولة العمليات الحسابية وكذلك استخدام أقل وقت وتخزين على الحاسوب .