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The Finite Element Technique in the Determination of Wing Flutter Speed of Aircraft

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ABSTRACT. The determination of flutter speed of aircraft is of a prime importance. This is done to ensure that the aircraft is flutter free within its flight envelope. The present method for the determination of flutter speed uses the finite element technique to calculate the mass and stiffness distribution of the whole structure of the aircraft and hence to compute the natural frequencies and the mode shapes of the complete structure. After adding the aerodynamic forces Theodorsen method is used to solve the flutter stability determinant, and the flutter speed is determined by the point of intersection of the real and imaginary root loci. The present method is used to determine the flutter speed of a commuter airliner and the results are given.

1. Introduction

The occurance of wing flutter within the flight envelope of an aircraft can hardly be other than catastrophy. It is, therefore, essential to ensure that the aircraft wing is "flutter free" before the aircraft can fly. Flutter investigations began as early as 1936[1], when the only available calculating machines were of the hand operated type. Due to this limitations the elastic structure of the aircraft was, in the solution, replaced by a semirigid structure having only two degrees of freedom. Routh's criterion was employed to examine the stability of the aircraft. An inverse method was later proposed[1], where an external force was added to the system and the values of flutter speed and flutter frequency were those which would vanish the added external loads. Although this method was limited due to the absence of advanced computers at that time it was recently brought back[2] to be used on small computers with 32K RAM.

The widespread of microcomputers with at least 640K RAM has made life easier and it would be more appropriate to use a more rigorous approach to solve an important problem such as wing flutter.

At present, the method commonly used is that of repeated direct solution of the eigenvalue problem where the aircraft speed is varied in steps until a flutter boundry is crossed[3-5]. This method is sometimes criticized because it treats the problem from mathematical view point, where physical insight is lost. However, Baldock[6] presented a method to enable the results to be understood in terms of the constituent systems.

On the other hand the accuracy of the method is restricted by the accuracy of the physical data used. This is why in this paper the finite element technique is used as a tool for the solution of the problem. The present work is based on a previous paper by Nahas[7].

2. Finite Element Models

An example representation for a commuter high wing aircraft is shown in Fig. 1

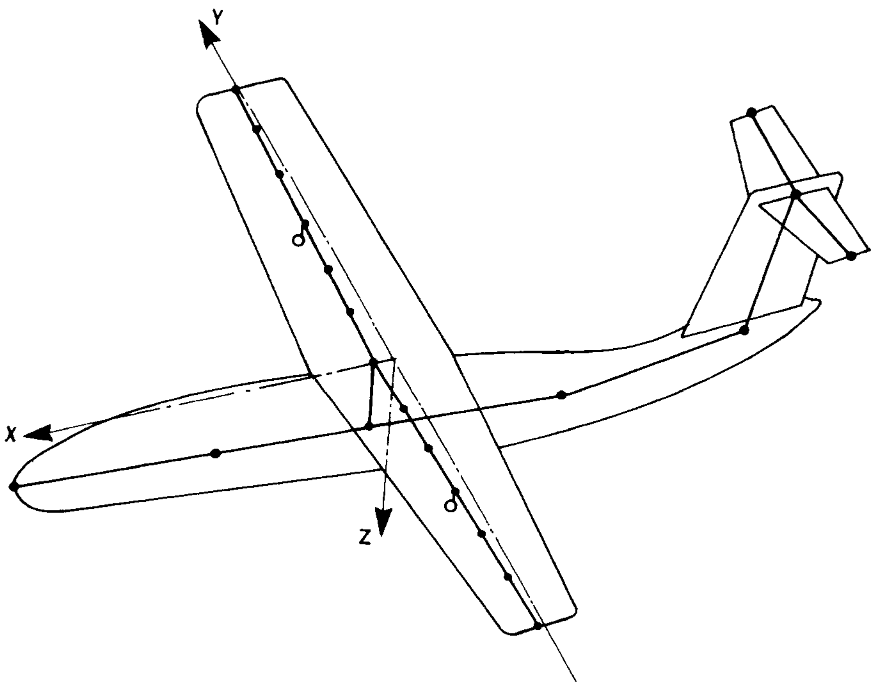

FIG. 1. Finite elements representing the aircraft.

where beam elements were used. To reduce the core required in the computer two different finite element models were used. The first was for the aircraft in the symmetric motion, Fig. 2, and the other was for the antisymmetric motion of the aircraft, Fig. 3. More elements were used to represent the wing than used to represent the other parts of the aircraft because wing flutter is the most critical case of flutter.

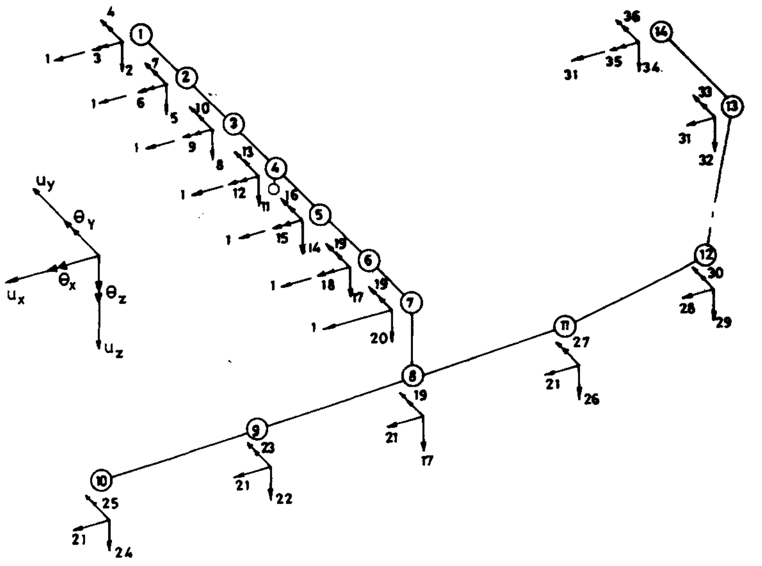

FIG. 2. Finite element model for the symmetric motion.

Six degrees of freedom (that is three rotations and three translations) at each node can be used, but, again, to reduce the memory required in the computer only the important freedoms were used in the models as shown in Fig. 2 and 3. Mazet[8] states

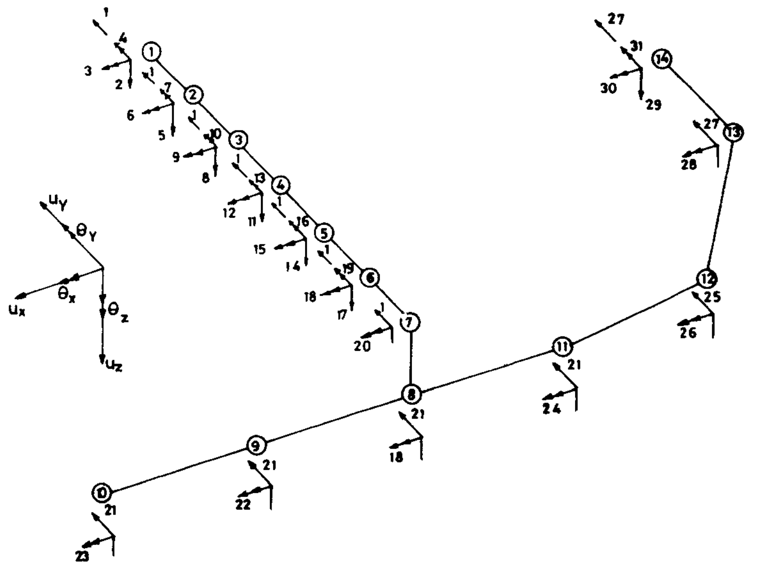

FIG. 3. Finite element model for the antisymmetric motion.

that the freedoms of roll, pitch and heave are the most important freedoms in the wing and the tailplane. For the fuselage and the fin in wing flutter problem the pitch and heave are important in the symmetric case, while the roll is the only important freedom in the antisymmetric case. However, to account for the difference in level between the datum of the different parts of the aircraft all the nodes were given an extra degree of freedom. This was the surge freedom in the symmetric model and the sway freedom in the antisymmetric model. This would imply that all nodes which have the same z-coordinate would have the same freedom number. The powerplant and the landing gears were idealized as lumped masses concentrated in their places. Similarly, non structural loads such as payload, fuel and furnishings were also idealized as masses but distributed in the appropriate locations.

To reduce rounding errors during the solution routine the sequence of the freedom numbering adopted was the node by node numbering fashion, taking into account, however, the restraints between the wing and fuselage by assigning the same freedom number where appropriate.

3. Mass and Stiffness Matrices

The overall mass and stiffness matrices for the whole structure were derived as usual from the individual element mass and stiffness matrices after transformation from element axes to global axes. The element mass and stiffness matrices require the determination of the mass of the element and its cross-sectional properties including the positions of the centre of mass and the flexural centre. A computerized procedure was employed to obtain all this information at many stations along each part of the aricraft. Real structural data were input to obtain exact properties. The data needed by the program included the stringer positions and areas, skin thicknesses, spar positions and thicknesses (for the wing and similar structures) and floor position and thicness (for the fuselage).

4. Natural Frequencies and Mode Shapes

The natural frequencies and the mode shapes were derived by organizing an eigenvalue problem from the mass matrix $[M]$ and the stiffness matrix $[S]$ by first writing Lagrange's equation

$$[M] \cdot \{U\} + [S] \cdot \{U\} = 0 \tag{1}$$

where $\{U\}$ is the vector of generalized coordinates, *i.e.*, the node freedoms of the problem.

When a harmonic solution is substituted in equation (1) the following eigenvalue equation is obtained

$$[S]\,\{U\} = \Omega^2\,[M]\,\{U\} \tag{2}$$

The eigenvalues Ω and the eigenvectors $\{\varnothing\}$ of equation (2) are respectively the required natural frequencies and mode shapes of the complete structure of the aircraft.

The diagonal mass and diagonal stiffness matrices are then obtained from the equations

$$\text{diag}\,[M] = \{\varnothing\}^{T} \cdot [M] \cdot \{\varnothing\} \tag{3}$$

$$\text{diag}\,[S] = \{\varnothing\}^{T} \cdot [S] \cdot \{\varnothing\} \tag{4}$$

5. Aerodynamic Forces

The aerodynamic forces are taken from Fung[4] and rearranged here in matrix forms to be consistent with the present method of solution. These forces are given for the two cases as follows

5.1 The Symmetric Case

$$\begin{bmatrix} L \\ P \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} h \\ \alpha \end{bmatrix} \tag{5}$$

where the coefficients A's are as follows

$$A_{11} = -\pi \rho V^2 [-k^2 + 2C(k)\, ik] \tag{6}$$

$$A_{12} = -\pi \rho V^2 b\,(ak^2 + ik) + 2C(k)\,[1 + ik\,(½ - a)] \tag{7}$$

$$A_{21} = \pi \rho V^2 b\,[2C(k)\, ik\,(½ + a) - k^2 a] \tag{8}$$

$$A_{22} = \pi \rho V^2 b^2 [2(½ + a)\, C(k)\, \{1 + ik(½ - a)\} + k^2/8 + k^2 a + (a - ½)\, ik] \tag{9}$$

where
- L is the lift force,
- P is the pitching moment,
- h is the vertical deflection,
- α is the pitch deflection,
- b is the semi-chord,
- a is the distance from mid-chord to elastic axis,
- V is the airspeed,
- ρ is the air density,
- k is the reduced frequency, $k = b\Omega/V$,
- i is the symbol of imaginary number,
- $C(k)$ is Theodorsen's function which is given by

$$C(k) = \frac{H_1^{(2)}(k)}{H_1^{(2)}(k) + i\, H_0^{(2)}(k)} \tag{10}$$

where $H_n^{(2)}(k)$ is Hankel's function of second kind and order n. Hankel's functions are given by Spiegel[9].

$$H_n^{(2)}(k) = J_n(k) - i\, Y_n(k) \tag{11}$$

where,

J_n is Bessel's function of first kind and order n and,

Y_n is Bessel's function of second kind and order n.

In terms of the generalized coordinates the aerodynamic forces are written as follows

$$\{Q\} = [F] \cdot \{U\} \tag{12}$$

where $\{Q\}$ is the vector of the aerodynamic forces and

$$F_{ij} = \int_0^s (A_{11} h_i h_j + A_{12} h_i \alpha_j + A_{21} h_j \alpha_i + A_{22} \alpha_i \alpha_j)\, dx \tag{13}$$

where s is the semispan.

5.2 The Antisymmetric Case

The aerodynamic forces in this case include the aileron moment and can be written as follows

$$\begin{bmatrix} L \\ P \\ R \end{bmatrix} = [A] \begin{bmatrix} h \\ \alpha \\ \beta \end{bmatrix} \tag{14}$$

where,
R is the rolling moment and,
β is the aileron deflection.

The equations of the coefficients A's are omitted here for brevity. The forces can also be written in terms of the generalized coordinates in a similar manner to the symmetric case, *i.e.*, as in equation (12) except that equation (13) becomes here as follows

$$F_{ij} = \int_0^s (A_{11} h_i h_j + A_{12} h_i \alpha_j + A_{13} h_i \beta_j + A_{21} \alpha_i h_j + A_{22} \alpha_i \alpha_j + A_{23} \alpha_i \beta_j + A_{31} \beta_i h_j + A_{32} \beta_i \alpha_j + A_{33} \beta_i \beta_j)\, dx \tag{15}$$

6. Flutter Equation and Flutter Determinant

The flutter equation in matrix form is[7]

$$[M] \cdot \{U\} + [S] \cdot \{U\} = \{Q\} \tag{16}$$

Substituting a solution for a simple harmonic motion in this equation the following equation is obtained

$$[-\Omega^2 M + S - F]\{U\} = 0 \tag{17}$$

For non-trivial solution the determinant of equation (17) must vanish, giving

$$|-\Omega^2 M + S - F| = 0 \tag{18}$$

This is the flutter determinant. However, with the presence of structural damping of coefficient g it becomes

$$|M(-\Omega^2 + i\Omega^2 g) + S - F| = 0 \tag{19}$$

7. Results

The present analysis was used to find the flutter speed of the commuter aircraft of Fig. 1. The structural details of the wing for this aircraft are given in Fig. 4 and Table 1. The flutter stability determinant of equation (18) was solved by assigning a value to the airspeed V and then finding the values of the real and imaginary parts of the determinant for a range of the frequencies. When ths sign of the real or imaginary part of the determinant is changed there must be a zero for that part, which was determined by interpolation. The airspeed was then increased and the routine was repeated to find other roots for the real and imaginary parts of the determinant. The roots were then plotted against the airspeed. Any point of intersection of the real and

TABLE 1. Wing structural data.

Bay	Upper Surface		Lower Surface		Spars			Leading Edge thickness (mm)
	skin thickness (mm)	stringer area (mm^2)	skin thickness (mm)	stringer area (mm^2)	thickness of front spar (mm)	thickness of rear spar (mm)	area of spar cap (mm^2)	
1-2	2.64	130	2.95	168	1.63	1.83	250	1.0
2-3	2.64	130	2.95	168	1.63	1.83	250	1.0
3-4	2.64	130	2.95	168	1.63	1.83	250	1.0
4-5	2.64	130	2.95	168	1.63	1.83	250	1.0
5-6	2.64	130	2.95	168	1.63	1.83	250	1.0
6-7	2.34	104	2.34	104	1.63	1.83	250	1.0
7-8	2.34	104	2.34	104	1.63	1.83	250	1.0
8-9	2.34	104	2.34	104	1.63	1.83	250	1.0
9-10	1.63	93	1.63	93	1.63	1.83	130	0.7
10-11	1.63	93	1.63	93	1.63	1.83	130	0.7
11-12	1.63	93	1.63	93	1.63	1.83	130	0.7
12-13	1.63	93	1.63	93	1.22	1.22	105	0.7
13-14	1.22	93	1.22	93	1.22	1.22	105	0.7
14-15	1.22	93	1.22	93	1.22	1.22	105	0.7
15-16	1.22	66	1.22	66	1.22	1.22	105	0.7
16-17	1.22	66	1.22	66	1.22	1.22	105	0.7
17-18	1.22	66	1.22	66	1.22	1.22	105	0.7

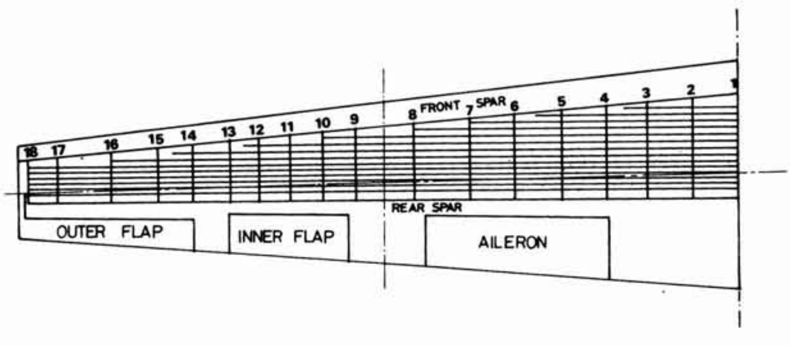

FIG. 4. Wing structural details of the aircraft.

imaginary root loci represents a root for the determinant, *i.e.*, it is a flutter point. The results obtained from the computer program are shown in Fig. 5 for the symmetric

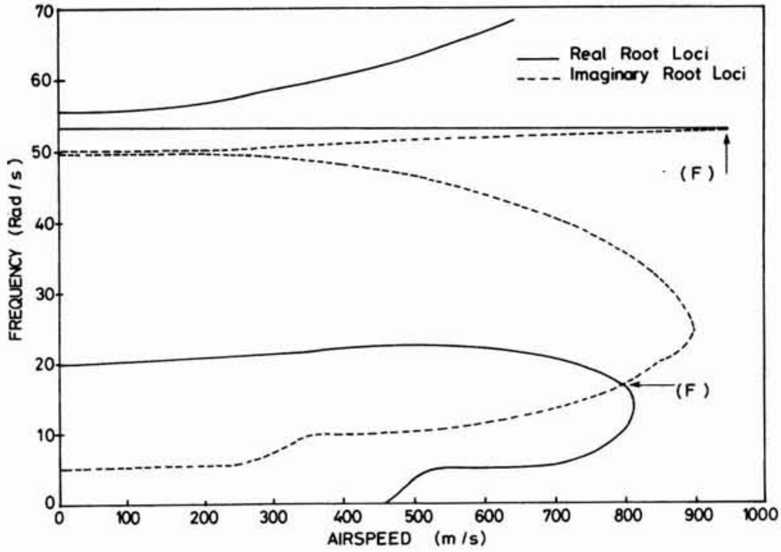

FIG. 5. Solution of the flutter determinant in the symmetric motion.

motion. The flutter points are denoted by (F) in the figure. The number of the flutter points equals the number of the degrees of freedom of the problem. The lowest flutter speed is taken as the flutter speed for the aircraft. Figure 6 shows the solution of the flutter determinant for the aircraft in the antisymmetric motion. The flutter

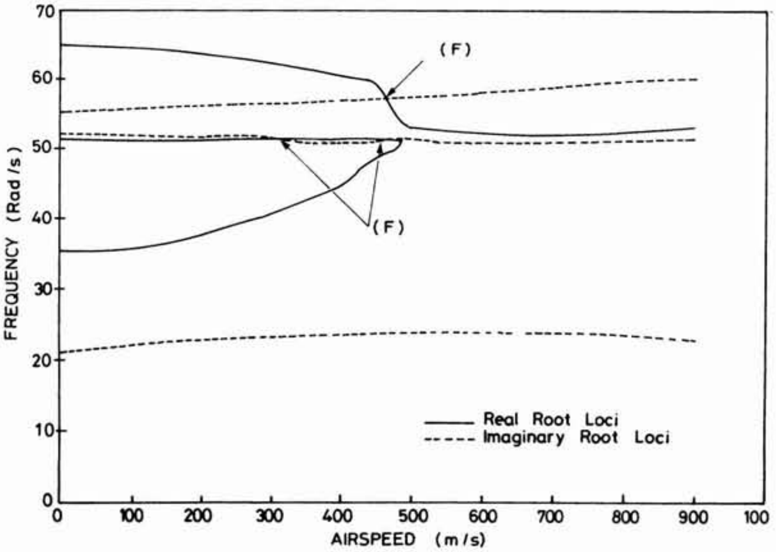

FIG. 6. Solution of the flutter determinant in the antisymmetric motion.

speed as read from Fig. 5 is 800 m/s, whereas it is 300 m/s in the antisymmetric case as read from Fig. 6. The diving speed for this particular aircraft is 154.5 m/s (or 300 knots). This ensures that the aircraft is flutter free.

8. Conclusion

The idea of employing finite element technique to solve the complicated problem of wing flutter has been proven to be possible and applicable. The present method takes the responsibility of doing all the calculations required to obtain the flutter speed of the aircraft without introducing any simplification to the structure. As many degrees of freedom as desired can be used provided the memory of the concerned computer is of enough size.

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استخدام طريقـة العناصر المتـناهية في تحديد سرعـة الطائرة عند رفرفـة الجناح

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جــــدة - المملكة العربية السعودية .

المستخلص . إن تحديد سرعـة الـرفرفة في الطائرة هي من الأهمية بمكان . ويتم ذلك للتأكد من أن الطائرة خالية من الرفرفة ضمن حدود غلاف الطيران . والطريقة المقدمة في هذا البحث لتحديد سرعة الرفرفة تستخدم طريقة العناصر المتناهية لحساب توزيع الكتل والجساءات لكامل إنشاء الطائرة ، وبالتالي لحساب الترددات الطبيعية وأنماط الاهتزاز لإنشاء الطائرة كاملاً . وبعد إضافة قوىٰ التحريك الهوائي تم استخدام طريقة ثيودورسون لحل مُعيِّن (مُحدِّد) استقرار الرفرفة ، وبالتالي تم تحديد سرعة الرفرفة من نقطة تقاطع المحلين الهندسيين للجذور الحقيقية والتخيلية . وفي الختام فإنه تم استخدام الطريقة الحالية في حساب سرعة الرفرفة لطائرة صغيرة وأعطيت النتائج في نهاية البحث .