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A FOUR-POINT NONLOCAL INTEGRAL BOUNDARY VALUE PROBLEM FOR FRACTIONAL DIFFERENTIAL EQUATIONS OF ARBITRARY ORDER

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Abstract

This paper studies a nonlinear fractional differential equation of an arbitrary order with four-point nonlocal integral boundary conditions. Some existence results are obtained by applying standard fixed point theorems and Leray-Schauder degree theory. The involvement of nonlocal parameters in four-point integral boundary conditions of the problem makes the present work distinguished from the available literature on four-point integral boundary value problems which mainly deals with the four-point boundary conditions restrictions on the solution or gradient of the solution of the problem. These integral conditions may be regarded as strip conditions involving segments of arbitrary length of the given interval. Some illustrative examples are presented.

Key words and phrases: Fractional differential equations; four-point integral boundary conditions; existence; Fixed point theorem; Leray-Schauder degree.
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1 Introduction

Boundary value problems for nonlinear fractional differential equations have recently been studied by several researchers. Fractional derivatives provide an excellent tool for the description of memory and hereditary properties of various materials and processes. These characteristics of the fractional derivatives make the fractional-order models more realistic and practical than the classical integer-order models. As a matter of fact, fractional differential equations arise in many engineering and scientific disciplines such as physics, chemistry, biology, economics, control theory, signal and image processing, biophysics, blood flow phenomena, aerodynamics, fitting of experimental data, etc. [17, 18, 19, 20]. Some recent work on boundary value problems of fractional order can be found in [1, 2, 3, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 22] and the