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Approximating common fixed points of two asymptotically quasi-nonexpansive mappings in Banach spaces

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Abstract

Suppose K is a nonempty closed convex subset of a real Banach space E . Let $S, T : K \rightarrow K$ be two asymptotically quasi-nonexpansive maps with sequences $\{u_n\}, \{v_n\} \subset [0, \infty)$ such that $\sum_{n=1}^{\infty} u_n < \infty$ and $\sum_{n=1}^{\infty} v_n < \infty$, and $F = F(S) \cap F(T) := \{x \in K : Sx = Tx = x\} \neq \emptyset$. Suppose $\{x_n\}$ is generated iteratively by $x_1 \in K, x_{n+1} = (1 - \alpha_n)x_n + \alpha_n S_n [(1 - \beta_n)x_n + \beta_n T_n x_n], n \geq 1$ where $\{\alpha_n\}$ and $\{\beta_n\}$ are real sequences in $[0, 1]$. It is proved that (a) $\{x_n\}$ converges strongly to some $x^* \in F$ if and only if $\liminf_{n \rightarrow \infty} d(x_n, F) = 0$; (b) if X is uniformly convex and if either T or S is compact, then $\{x_n\}$ converges strongly to some $x^* \in F$. Furthermore, if X is uniformly convex, either T or S is compact and $\{x_n\}$ is generated by $x_1 \in K, x_{n+1} = \alpha_n x_n + \beta_n S_n [\alpha'_n x_n + \beta'_n T_n x_n + \gamma'_n z'_n] + \gamma_n z_n, n \geq 1$, where $\{z_n\}, \{z'_n\}$ are bounded, $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}, \{\alpha'_n\}, \{\beta'_n\}, \{\gamma'_n\}$ are real sequences in $[0, 1]$ such that $\alpha_n + \beta_n + \gamma_n = 1 = \alpha'_n + \beta'_n + \gamma'_n$ and $\{\gamma_n\}, \{\gamma'_n\}$ are summable; it is established that the sequence $\{x_n\}$ (with error member terms) converges strongly to some $x^* \in F$. © 2006 N. Shahzad and A. Udomene.

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