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Income Ratio, Risk-Sharing, and the Optimality of *Mudarabah*

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Abstract. Little attention in the current literature is given to the analysis of interest-free Islamic financing tools within the framework of risk-return portfolio analysis. This paper adopts the standard tools to establish interesting optimal properties of mudarabah within a two-party contractual model of an income generating economic activity. 'Income ratio' is defined as the percentage of expected income that goes to each of the two parties whereas the 'risk-sharing structure' refers to how risk is shared between the two parties. The key question is how these two parameters are related through alternative two-party contracts. This paper is an extension to another one where the risk-sharing structure is shown to be perfectly proportionate to income ratio in the case of mudarabah. The *Shariah*-prohibited interest-rate financing is a manifest disproportionate case as the income ratio of lender is totally insensitive to the contract's risk.

Adopting the same competitive set-up within an informational efficient environment, this paper sets out to establish two more findings: First, a negative relationship proves to exist between income ratio and the risk-sharing structure in terms of an *optimal contracts curve* (OCC). The mudarabah contract emerges at an optimal break-even point where the OCC coincides with pure profit-sharing but fixed return financial leverages also co-exist with mudarabah. Hence, a pure equity-based Islamic order is theoretically inconceivable even under ideal information efficiency. Second, the optimal mudarabah income ratio (*i.e.* profit-sharing ratio) is shown to depend crucially on the extent to which the two parties differ in their attitude towards risk (*i.e.* the risk-attitude differential). The paper goes further to examine the impact of risk-attitudinal differentials on the optimal profit-sharing ratio. These findings are shown to have useful practical and policy implications.

1. Introduction

Profit-sharing through mudarabah financing is believed to be the genuine financing alternative to the forbidden interest rate system in Islamic economics. The current interest in the theoretical dialogue about optimal financial contracts in the Islamic perspective can be traced back to an initial work by Khan (1985) who established a Pareto-optimal theorem for mudarabah when compared to fixed interest rate financing under conditions of informational symmetry. Khan theorem, however, suffered from a fundamental weakness as it implied mudarib’s risk neutrality even though the author argued otherwise (Tag el-Din, 1990). Nonetheless, Khan’s theorem was a landmark in the literature of Islamic economics in bringing to a sharp focus the issue of informational asymmetry in the mudarabah contract and its consequent moral hazard problem. Ever since, increasing interest in the theory of optimal contracts and incentive-compatible systems for profit-sharing systems followed suit, as exemplified by Haque and Mirakhor (1987), Tariquallah Khan (1995), Bashir (2001), and Ahmed (2002). More recently, various contributions with regard to the moral hazard problem have been made in Iqbal (2001), and Iqbal and Llewellyn (2002) by Balkhail and Presley (2002) as well as Khalil *et al*, (2002).

A fundamental point that seems to underlie these studies is hypothesis that the mudarabah contract would dominate over prefixed return contracts had it not been for informational asymmetry. It is tantamount to saying the mudarabah contract should dominate over prefixed rate financing within a theoretical model of informational symmetry. This hypothesis however remains to be thoroughly examined if the current concerns with incentive-compatible schemes in mudharabah have to make real sense. The common feature of all incentive-compatible schemes has been to address the principal/agent problem arising from informational asymmetry in the *mudarabah* contract between financier (*rabb al-mal*) and financee (the *mudarib*). While such an approach is vitally needed to offer practical solutions to the information asymmetry problem, the question remains as to whether the claimed optimality of *mudarabah* is sustainable under the assumption of information efficiency. This also relates to the fundamental question whether or not an ideal Islamic economy should be purely equity-based as opposed to the current order where fixed return financial tools (*murabahah, ijarah, etc.*) tend to dominate the present scene of the Islamic financial industry.

Little attention seems to have been given to the risk-return optimality of mudarabah under informational symmetry except perhaps for Tag El-Din (1992, 2002) where a Risk-Return Sharing Model (RRSM) has been introduced as a special version of an Edgeworth box with two risk-averse parties (capital provider and manager) within an ex ante informational efficient environment.

The RRSM presents the problem of financial choice as one involving three possible options: a pure *variable* return (through mudarabah profit sharing), a pure risk-free *fixed* return and a combination of the two. In particular, the mudarabah contract emerges as a special case of an optimal contract whereby risk is shared in the same ratio as income. Given the assumption of *expected utility maximization*, the objective of this paper is to explain the pattern of various possible risk-sharing structures embodied by optimal two-party contracts and the position of mudarabah in this context, and how it is affected by different assumptions about risk-aversion.

The beginning of modern portfolio theory dates back to 1952 when Harry Markowitz introduced the concept of a mean-variance efficient frontier for a set of investment securities. However, the theory acquired its computational convenience mainly through the 'single index model', introduced in 1963 by William Sharpe. Subsequent theoretical refinements and practical developments led to the formulation of the Capital Assets Pricing Model (CAPM) by Sharpe (1985), Litner, and Mossin. In a nut-shell, the CAPM presents the efficient portfolio as a single combination of risky securities (The market portfolio) augmented by borrowing and lending along a Capital Market Line. The CAPM continues to maintain its recognition in the standard textbooks of finance, and the advanced computerized packages of financial analysis, notwithstanding the severe theoretical criticisms in the current literature.

Attitude towards risk is the decisive element of the whole exercise. Admittedly, the whole risk-return structure will boil down to pure mathematical tautology unless it relates to an economically consistent scale of preference. A typical investor is, thus, assumed to select his efficient portfolio in terms of a convex-downwards risk-return indifference curve. Tobin makes the assertion that for normally distributed returns the convexity property must necessarily hold (Tobin 1974), Feldstein (1969), Borch (1969). The basic implication of the CAPM that seems to contradict Islamic economics is the presentation of capital market lending and borrowing at a fixed risk-free interest rate as an integral part of an efficient portfolio. This point has been addressed by Tag El-Din (1990) through the questioning of Tobin's assertion that investors' risk-return indifference curves must be convex from below when investment returns are normally distributed. Nonetheless, it has been shown through the RRSM that the convexity property presents mudarabah financing as a Pareto-optimal contract.

2. Basic Background

The underlying assumptions of investment portfolio theory are fully detailed in Haugen (1986, pp: 155-184). While utilizing basic assumptions of the mean-variance investment portfolio theory, it will shortly be explained that the objective of this analysis is quite distinct from portfolio theory. The study departs from the usual free market competitive conditions in addition to the following two basic assumptions:

- *Informational efficiency*: Given the uncertain financial environment, the subject matter of financial market information are the return and risk parameters as they are measured in terms of mean (q) and standard deviation (s). Informational efficiency is therefore the knowledge of these two parameters by all potential participants.

- *Risk-aversion:* A risk-averse investor is one who trades return for risk. As in the standard practice, risk-aversion is represented by a concave utility function, which amounts to convex indifference curves from below within the 'return-risk' space. Investors are, hence, assumed to maximize utility within the return-risk space in the sense of seeking higher income ratios at lower risk-sharing ratios.

The query about the optimal risk-return contractual relationship will never arise if the parties were risk-neutral. If risk-neutrality prevails it is only income ratio that matters but when both parties are risk-averse the risk-sharing structure does matter. Each party is then assumed to seek maximum expected utility, in the sense *of maximum possible income share coupled by minimum possible risk share*. This is in fact the basic assumption which underlies the shape of the optimal contracting curve (OCC) in the subsequent analysis.

Incidentally, the assumption of risk-aversion is also implicit in the risk-sharing jurisprudence of mudarabah. In fact, it is the very recognition of risk as an undesirable reality of economic life that underlies the Islamic concept of justice. The idea is to share *undesirable* risk fairly between the parties rather than throw it disproportionately on other parties. Admittedly, there is more to the jurisprudence of mudarabah than what can be derived through the utility maximization assumption. It is possible from a jurist viewpoint that one of the two parties behaves charitably, hence, accepts a very small share, or even no share at all in the mudarabah profit[(1)]. But, altruism is not the general rule in the financial market. It is still interesting to see how possible it is for expected utility maximization to yield an ethical result if the ethical rule in market is 'fairness' rather than charitableness.

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(1) The idea of *ibda'* is the case in point where all profit is donated by the *mudarib* to the financing party (*rabb al-mal*) (al-Mawsu`ah al-Fiqhiyah (1993), pp: 172-178).

2.1 Income Ratio and Risk-Sharing

The term 'income' stands in this paper for net income that is expected to be generated and immediately distributed to the two parties of an economic activity: capital provider (**A**) and manager (**B**). The term 'income ratio' is then used to denote the division of such expected income between the two parties regardless of how risk is shared between them. Rather than 'profit', 'income' is the relevant term within the present context since it allows for the comparison of fixed income agreements (*e.g.* **B** paying fixed interest to **A,** or **A** paying fixed salary to **B**) with alternative profit-sharing agreements. From a strictly accounting perspective, fixed interest or salary payments are not treated as shares in 'profit'; hence, to view the income percentage of such fixed liabilities it is more appropriate to speak of 'income ratio' rather than 'profit ratio'.

This approach will therefore make it possible to compare **A's** share in the mudaraba profit as *rabb al-mal* with **A's** share in income as *lender*, even though the former is an uncertain share in profit while the latter is a guaranteed repayment of principal plus interest. In other words, **A**'s *income ratio* can be equal in both *rabb al-mal* and *lender* capacities even though the associated risk is manifestly different. More contractual forms are still possible to arise from the division of income between the two parties. For example, a risk-free salary for manager **B** places the entire burden of risk on the capital provider **A** as in the employer/employee model. Hence, fixed salary can also act as the basis of income ratio. Other hybrid contractual combinations of profit-sharing are demonstrated in this paper with either fixed interest or fixed salary.

Without loss of generality, the income ratio, *e.g.* $a_0 = 0.25$, is regarded as **A**'s fractional share in the expected disposable income such that **B**'s share is the remaining fraction $(1\text{-}a_0) = 0.75$. In the mudarabah contract, $a_0 = 0.25$ takes the form of the profit-sharing ratio that is promised to party **A** as *rabb a-mal* against the remaining fraction $(1\text{-}a_0) = 0.75$ to party **B** as the *mudarib*. Equally well, it may refer to a borrowing contract where a fraction $a_0 = 0.25$ of the excepted income representing principal plus interest is guaranteed to party **A** as lender against a non-guaranteed payment of $(1\text{-}a_0) = 0.75$ to party **B** as manager. Note that 'expected' income is a known fixed number q at the time of contracting even though it is an uncertain quantity. Thus, if the expected income is q = £400, then party **A** will expect a return of a_0q = £100 regardless of how risk is shared, and similarly party **B** will expect a return of $(1\text{-}a_0)$ q = £300 under both contracts regardless of how risk is shared.

Risk-sharing comes into play when the purpose is to qualify on how guaranteed or otherwise are the expected returns of the two parties. The standard deviation of the expected income s is normally used as a measure of risk. In the above example, if **A**'s £100 return bears a proportionate share in risk (*i.e.* 0.25s), then **B**'s £300 also bears its proportionate share in risk (*i.e.* 0.75s). This situation of proportionality has been shown to be the mudarabah case (Tag el-Din, 2002). Alternatively, if **A**'s return of £100 is contractually guaranteed (*i.e.* s = 0), **B's** return of £300 will bear all the contractual risk. Obviously, this is the Shariah-prohibited interest rate financing where **A** is guaranteed a fixed return in the contract regardless of risk.

2.2 The Two-Party Model

The model assumes two contracting parties, **A** and **B**, aiming respectively to provide finance and management for an income-yielding project. All finance is provided by party **A** (the capital provider) while all management is provided by party **B** (the manager). The basic model can be expressed in terms of one random variable **X** to represent the distributable expected income with parameters: $E(X) = \theta$ and $Var(X) = \sigma^2$.

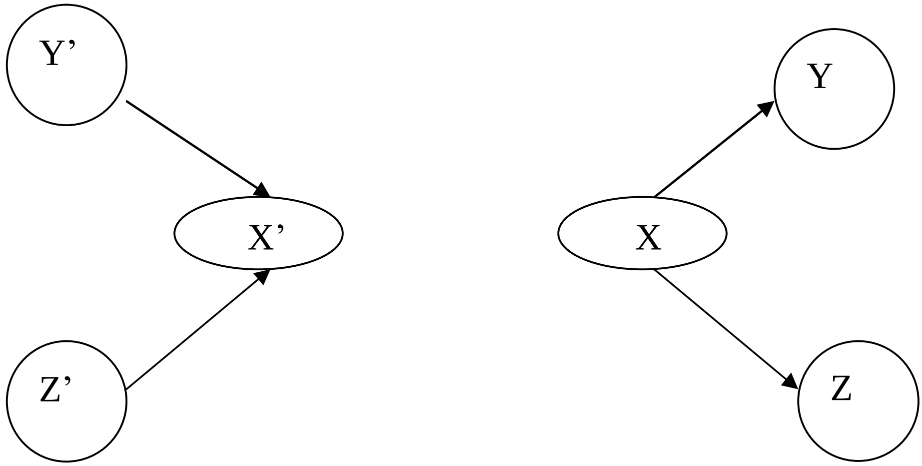
As regards the share of the two parties in the total income **X** in terms of any fixed income ratio a_0 this is represented by the random variables, **Y** and **Z**, as below:

$Y = \alpha_0 X$, and $Z = (1-\alpha_0)X$

Such that: $E(Y) = \alpha_0\theta$ and $E(Z) = (1-\alpha_0)\theta$; for all $0 < \alpha_0 < 1$.

The two variables Y and Z are perfectly dependant since both are defined in terms of the same independent income variable X. It is noteworthy that X is the only investment asset with parameters $E(X) = \theta$ and $Var(X) = \sigma^2$, and the whole exercise is to show how the parameters θ and σ break up between the two parties **A** and **B** upon dividing X into two share variables Y and Z.

Obviously, the two-party model is not a problem in investment portfolio analysis where two investment assets Y' and Z' are combined to make up for the portfolio X'. This is just the reverse of the two-party model since the portfolio X' is strictly dependent on the assets Y and Z, and the objective is to see how the risk-return parameters of the two assets Y', Z' make up for the parameters of portfolio X'. In particular, when Y' and Z' are statistically dependant, the risk of the portfolio X' (*i.e.* the square root of Var (X')) turns out to be a complex variance/ covariance formula; this contrast is shown in figure [1], (a), (b) below.



(a) **Investment portfolio analysis**: two assets Y' and Z' make up the portfolio X'

(b) **The Two-party model** Single asset X divided into two shares Y and Z

Fig. (1). Y', Z' as ingredients of portfolio X' contrasted with Y, Z as shares out of income X.

The crux of the matter is to define the share variable **Y** and **Z** with a view to the various risk-sharing structures that may arise under different possible contracts between the two parties. For this purpose the risk-sharing parameter β is deliberately introduced as:

$$0 \leq \beta \leq 1$$

Then the idea is to show, for any given income ratio $a = a_0$, how total risk s is proportionately or disproportionately shared by the two parties under different possible contracts.

3. Five Possible Risk-Return Contracts

The above *information efficiency* assumption implies that the parameters (θ,σ) are known fixed numbers to both parties at the time of contracting. Hence, given the fixed income ratio a_0 and the above definition of the risk sharing parameter b there are five possible risk-sharing schemes between the two contracting parties as follows:

1. *The Mudarabah Contract*: This is the case where party **A** as *rabb al-mal* and **B** as the *mudarib* share the risk s strictly proportionately to their income ratios. That is, b **=** a_0 and **(1-**b**) = (1-**a_0**)** respectively. This property follows directly from:

   $E(Y) = \alpha_0\theta$ and $Var(Y) = \alpha_0^2\sigma^2$, and
   $E(Z) = (1 - \alpha_0)\,\theta$ and $Var(Z) = (1 - \alpha_0)^2\,\sigma^2$
   Thus, the ‘‘return, risk’ parameters of the mudarabah parties are given as: (a_0q**,** a_0s) for A and **[(1-**a_0**)**q**, (1-**a_0**)**s**]** for **B**.

2. *The pure Borrowing model*: This is the case where party **A** is a lender against an expected guaranteed payment **r** consisting of repaid principal and interest**. A** bears zero contractual riskl[2]; that is, b **= 0**. On the other hand, party **B** is the borrower who reaps the remaining expected income **X -r**, but bears all the risk s ; that is **(1 -** b**) = 1**. To represent this case, the two return variables are simply defined as:

   - $Y = r$
   - $Z = X\text{-}r$

   Bearing in mind that q is a known number at the time of contracting, the constraint of fixed income ratio a_0 and (**1-** a_0) is satisfied for the two parties as:

   - $\alpha_0\theta = E(Y)$
     $= r$, with $Var(r) = 0$, which is equivalent to:
   - $(1- \alpha_0)\,\theta = E(Z)$
     $= E(X - r) = \theta - r$, with $Var(X - r) = Var(X) = \sigma^2$

   The ‘risk, return’ parameters for the two parties are respectively **(**a_0q**, 0**), and **[(1-**a_0**)**q**,** s **].** This is the most manifest case where risk s is disproportionately shared between two parities.

3. *Partial borrowing Contract*: This is also a case of disproportionate division of risk where party **A** bears a smaller share in risk than his share in income; that is b **<** a_0. More precisely, it is the contract where **A** is partially a lender with guaranteed fixed return, **r**, and partially an equity holder with a share b in the expected income. It conforms to *financial leverage* in the current terminology of corporate finance.

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(2) Note that default risk is not a contractual property and therefore it is ignored in the paper.

Conversely, **B** is partially a borrower at a fixed cost, **r**, and partially an equity holder with share (**1-**b**)** in the expected income, hence bearing a bigger share in risk than the income ratio *i.e.* (**1-**a_0**)** < (1-b**)**. The two random variables Y and Z for the two parties are hence expressible as:

- $Y = r + \beta X$
- $Z = (1-\beta)\, X - r$

Then to satisfy the constraint of fixed income ratio a_0 and (**1-**a_0), it follows that:

- $E(Y) = r + \beta\theta$
  $= \alpha_0\theta$
- $E(Z) = (1-\beta)\theta - r$
  $= (1-\alpha_0)\,\theta$

This leads immediately to the property $b < a_0$. Incidentally, it is also true that:

- $\mathrm{Var}(Y) = \beta^2\sigma^2$
- $\mathrm{Var}(Z) = (1-\beta)^2\,\sigma^2$

Hence, the 'risk, return' parameters for the two random return variables **Y, Z** are respectively **(**a_0q **,** b s**)** and **[(1-**a_0**)** q**,** **(1-**b**)**s**))**]. Notably, although the two parties continue to share expected income with the same fixed income ratio a_0, yet risk is disproportionately shared.

4. *The Pure Hiring Contract*: In this case, party **A** hires party **B**'s management services against a guaranteed salary, **s.** Contractually, **B** bears no risk at all; that is, (**1-**b**) = 0**. On the other hand, party **A** reaps the remaining expected income **X - s**, and hence bears the entire risk s ; that is b**=1**. To represent this case, the two return variables are simply represented as:

- $Y = X - s$
- $Z = s$

The constraints of fixed income ratio a_0 and (**1-** a_0) are satisfied for the two parties as:

- $\alpha_0\theta = E(Y)$
- $= E(X) - s = \theta - s$, and $\mathrm{Var}(X - s) = \sigma^2$
- $(1-\alpha_0)\,\theta = E(Z)$
  $= s$, with $\mathrm{Var}(s) = 0$.

The 'risk, return' parameters for the two parties are respectively **(a_0q, s^2)** and **[(1- a_0)q, 0].** This is another manifest case where risk s is disproportionately divided between the two parties.

5. *Partial hiring Contract*: This is a case of disproportionate division of risk where **B** bears a smaller share in risk than the share in income; that is (1-b**) < (1-**a_0**)**. More precisely, it is the contract where **B** is partially hired at a guaranteed fixed salary, **s**, and partially an equity holder with a share (1-b**)** in the expected income. It conforms to *operational leverage* in the current terminology of corporate finance.

Conversely, **A** is partially a hirer of management service at a cost, **s**, and partially an equity holder with a share b in the expected income, hence bearing a bigger share in risk than the income ratio; that is b > a_0. The two random variables Y and Z are expressible for the two parties as:

- $Y = \beta X - s$
- $Z = (1 - \beta) X + s$

Then to satisfy the fixed income ratio constraint a_0 and (**1-** a_0) for the two parties, it follows that:

- $E(Y) = \beta\theta - s$
  $= \alpha_0 \theta$
- $E(Z) = (1-\beta)\theta + s$
  $= (1-\alpha_0)\theta$

This leads immediately to the property b > a_0**.** However, it is also true that:

$$\mathrm{Var}(Y) = \beta^2\sigma^2$$
$$\mathrm{Var}(Z) = (1-\beta)^2\sigma^2$$

Hence, the 'risk, return' parameters for the two random return variables **Y, Z** are respectively **(**a_0q **,** b s**)** and **[(1-**a_0**)** q**, (1-**b**)**s**))**].

3.1 Representation of contracts within the RRSM

The above five contracts are now shown within RRSM involving the two parties **A** and **B** as they are placed on opposite sides of the box as in Edgeworth Box. Without loss of generality, the analysis is viewed from party **A**'s perspective along the regular return-risk axes, although it also applies by symmetry to party **B** along the rotated axes. And since the income-ratio a = a_0 is the same for all the five possible contracts, the latter are now shown along the same horizontal income ratio line, $a = a_0$ regardless of risk. It follows that b is the only parameter that distinguishes between the five contracts. In particular, the above property $b = a_0$ of the mudarabah contract gives rise to a '*Mudarabah* Line' that runs diametrically from the north-eastern corner to the south west corner of the box.

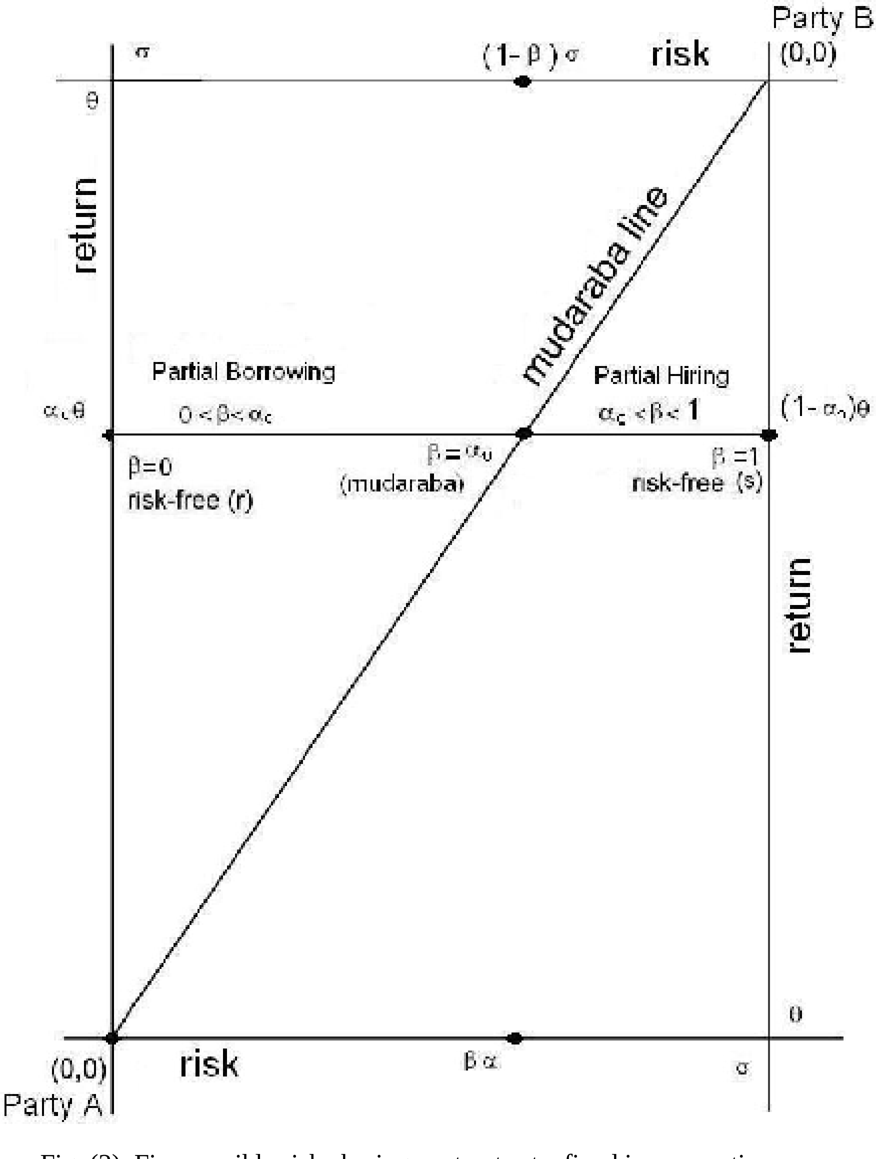

Fig. (2). Five possible risk-sharing contracts at a fixed income ratio $a = a_0$.

3.2 The Optimal Risk-Sharing Structure

To explore the risk-sharing structures which satisfy Pareto-optimality(3) for all possible income sharing ratios, an optimal contract curve (OCC) will be drawn to match any given income ratio a_0 with its optimal risk-sharing ratio b. In other words, it a question about the appropriate shape of the OCC which reflects the work of competitive market forces on the two contracting parties: managers as against capital providers, each seeking maximum expected utility within the risk/return space.

More formally, the required OCC is the geometrical representation of the set of points within the (q, s) space where marginal rates of substitution (MRS) in terms (q, s) are equal for the two parties. The OCC is, hence, the line which connects all tangential points formed within the RRSM by the two opposite families of indifference curves. Figure (3) below shows how the convexity property of upwards sloping (q,s) -indifference curves justifies a negatively sloping OCC. This is simply the analogue of the standard positively sloping 'contract line' within Edgeworth Box which reflects the convexity property of downwards sloping consumer indifference curves.

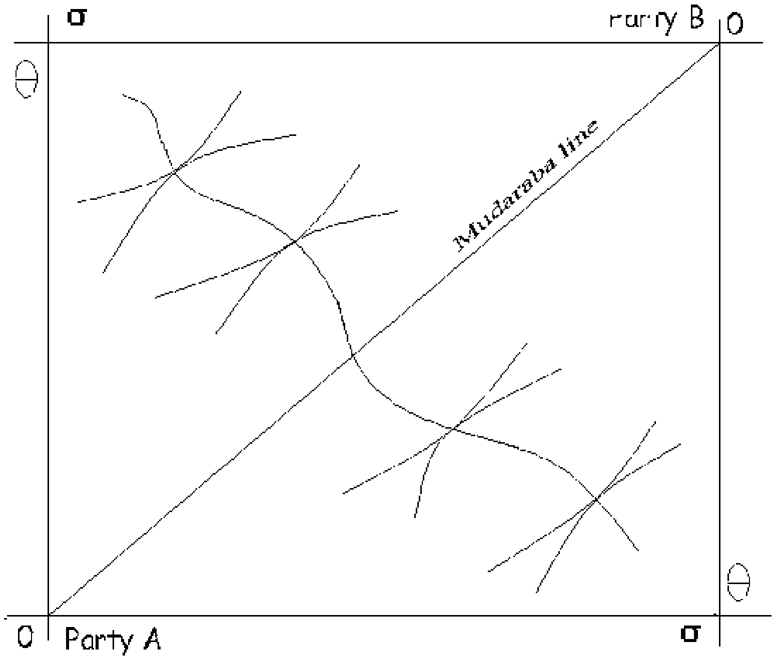

Fig. (3). The general shape of the OCC.

(3) That is, where no one party can made be better off without making the other party worse-off.

One way to test the above property of the OCC and run further analysis about the impact of risk-aversion differentials is to adopt a simple quadratic model for the above two-party model. The quadratic utility function is a familiar tool that is widely adopted in the current literature of portfolio investment theory due to its simple analytical appeal. Its major limitation is the failure to yield positive marginal utility beyond a certain region of the return variable, which is usually avoided by restricting the utility function to the domain where marginal utility is non-negative.

4. The Optimal Contract Curve

To examine the shape of the OCC, we shall adopt quadratic utility functions $\mathbf{U_A(Y) = a_0 + a_1Y + a_2Y^2}$, and $\mathbf{U_B\,(Z) = a_0 + a_1Z + a_2Z^2}$ for the two parties **A** and **B**, respectively, where $\mathbf{X = Y + Z}$ is defined above. The expected utility functions for the two parties A and B turn out to be:

- $U_A(\theta_1 , \sigma_1) = a_0 + a_1\theta_1 + a_2\,(\theta_1^2 + \sigma_1^2)$,
- $U_B((\theta_2 , \sigma_2) = b_0 + b_1\theta_1 + b_2\,(\theta_2^2 + \sigma_2^2)$, [1]

Where $\theta_1 = aq$, $s_1 = bs$ relate to Party A, while $q_2 = (1\text{-}a)q$ and $s_2 = (1\text{-}b)s$ relate to Party B. Total return $q = q_1 + q_2$ and total risk and $s = s_1 + s_2$ are fixed constants since they must be taken as given in the process of utility maximization. It is also noteworthy that $\mathbf{a_0}$ and $\mathbf{b_0}$ are arbitrary constants.

The quadratic utility function is known for the limitation that it cannot yield positive marginal utility beyond a certain region of the return variable. Hence, to guarantee positive marginal utility of expected returns for both parties, we must restrict utility functions to regions defined by the two conditions:

$a_1 + 2a_2\,\theta_1 > 0$, and $b_1 + 2b_2\,\theta_2 > 0$, [2]

Positive marginal utility necessitates $\mathbf{a_1 > 0}$ and $\mathbf{b_1 > 0}$. The concavity of the utility functions requires that the second order derivatives of the utility function are negative ($\mathbf{a_2 < 0}$ and $\mathbf{b_2 < 0}$), where:

$a_2 = \frac{1}{2}\,\partial^2 U_A /\partial\sigma^2_1 < 0$ and $b_2 = \frac{1}{2}\,\partial^2 U_B /\partial\sigma^2_2 < 0$, [3]

The two parameters $\mathbf{a_2}$ and $\mathbf{b_2}$ are instrumentally important in the subsequent analysis since they are the measures of risk-aversion for the two parties A and **B**, respectively. For example, Party A will be *less risk-averse* than Party **B** only if $\mathbf{a_2 > b_2}$.

Given the fixed $(q,\ s)$ values, the OCC for the two parties is defined in terms of the income and risk sharing values a , b which maximize one party's utility function subject to any level of the other's utility. The constrained utility function can be defined symmetrically for either of the two parties without affecting the result. For party **A** it is:

$$U^{*} = U_{A}(\theta_1, \sigma_1) + \lambda[U_{B}((\theta - \theta_1, \sigma - \sigma_1) - U^{(0)}_{B}]$$

Where l is the Lagrange multiplier, and $\mathbf{U^{(0)}_{B}}$ is an arbitrarily fixed level of party **B**'s utility function. The first order conditions of constrained maximization over the space of $\mathbf{(q_1,\ s_1)}$ are:

$$\begin{aligned}
\partial U^{*}/\partial\theta_1 &= \partial U_{A}/\partial\theta_1 - \lambda\partial U_{B}/\partial\theta_2 = 0\\
\partial U^{*}/\partial\sigma_1 &= \partial U_{B}/\partial\sigma_1 - \lambda\partial U_{B}/\partial\sigma_2 = 0\\
\partial U^{*}/\partial\lambda &= U_{B}(\theta - \theta_1, \sigma - \sigma_1) - U^{(0)}_{B} = 0
\end{aligned}$$

Then, the condition of Pareto optimality is given as:

$$(\partial U_{A}/\partial\theta_1)/(\partial U_{A}/\partial\sigma_1) = (\partial U_{B}/\partial\theta_2)/(\partial U_{B}/\partial\sigma_2) \qquad [4]$$

5. Occ and Mudarabah Break-Even Point

To derive the OCC on the basis of quadratic utility function, equation [4] above reduces to:

$$(a_1 + 2a_2\theta_1)/\ 2a_2\ \sigma_1 = (b_1 + 2b_2\theta_2)/\ 2b_2\ \sigma_2 \qquad [5]$$

Then, to focus primarily on risk-aversion rates and ignore unnecessary differences between the parties' utility functions we may simplify the above formula by letting $a_1 = b_1 = c$, leading to:

$$(c + 2a_2\theta_1)/\ 2a_2\ \sigma_1 = (c + 2b_2\theta_2)/\ 2b_2\ \sigma_2 \qquad [6]$$

Based on equation [6] above, the break-even theory of mudarabah within a competitive market can be described under various patterns of risk-aversion rates. The simplest case is the one where the two parties are equally risk-averse, and this is represented by the restriction $a_2 = b_2 = d$. Hence, under the special case of equally risk-averse parties, it is possible to establish the following properties (see Appendix for proof):

1. In agreement with the general form of the OCC as described in Figure [2] above, there exists a negative relationship between the income ratio α and the risk-sharing parameterβ. Interestingly, the quadratic form yields a linear relationship of the form:

$$\alpha = m + w\beta \qquad [7]$$

Where m = - c/2dθ , and w = (c + dθ)/dθ. In other words, α and β are proved to be negatively related such that the higher the income ratio for any party, the smaller the risk-share of that party. Conversely, the smaller the income ratio of any party, the greater is his risk-share. This property confirms the action of competitive market forces by the two utility maximizing agents, each seeking a higher income share at a lower risk share. The party with more bargaining power is able to achieve both objectives: more return share and less risk share. Yet, the weaker party will be obliged to accept a smaller income share and a bigger share in risk.

2. That the boundary contracts *pure borrowing* and *pure hiring* contracts are not represented on the OCC. This follows from [7] where :

$$0 < \beta < 1, \quad \text{for} \quad \text{all} \quad 0 \leq \alpha \leq 1,$$

Thus, the OCC accommodates only the contracts with definite risk-sharing structures (*partial borrowing*, *partial hiring*, and *mudarabah*) as defined above.

3. That the immediate effect of a negatively sloping OCC is to yield the mudarabah contract at the break-even point where the relative competitive market forces are fairly balanced. In the current case of equally risk-averse parties, the mudarabah contracts are located at the center of gravity (½, ½) of the RRSM, where the contracting curve intersects the positively sloping profit-sharing line. The optimal profit-sharing ratio for mudarabah financing turns out to be $\alpha = ½$, giving equal income shares for the two parties.

Two ethical distribution properties seem to be associated with mudarabah financing: *fair risk-sharing* and *fair income ratio*. The last point is particularly interesting, as it is not recognized in the current literature.

Next, it is interesting to see how the alternative situation of ‘unequal’ risk-aversion may affect the above property of equal income shares in mudarabah financing. Obviously, any change in the relative position of OCC should affect the optimal profit-sharing of mudarabah.

5.1 Unequally Risk-Averse Parties

It is easy to see that if one party is risk-averse and the other is risk-neutral, then the OCC will totally coincide with any one of the two boundaries of the model, leading to either *pure borrowing* model or *pure hiring* model. For example if party **B** (the manager) is risk neutral while party **B** (the financier) is risk-averse, the OCC will coincide with the pure fixed interest rate boundary for party **A**, leading to a pure borrowing model.

By symmetry, less restrictive results can be expected for the less restrictive case of *relative risk neutrality* where one party is closer to risk neutrality (less risk-averse) than the other. In the latter case, the OCC will come closer to a boundary rather than coincide with it. In general, we should expect the OCC to lie closer to the fixed return boundary for one party, the closer to risk-neutrality is the other party. Using the central point of the model (½ , ½) as bench mark, if the OCC is positioned to the left of (½, ½), then it must be closer to the fixed interest boundary. If it is positioned to the right of (½, ½), then it must be closer to the fixed salary boundary.

Then, to account for different attitudes towards risk, we shall define the risk attitudinal differential (RAD) in terms of the risk aversion parameters $\mathbf{a_2}$ and $\mathbf{b_2}$ with reference to [6], as :

$$\text{RAD} = a_2 - b_2 \qquad [8]$$

Notably, RAD > 0 implies that party A is closer to risk neutrality than party **B**, while RAD < 0 implies that party **B** is the closer to risk neutrality. We have just seen that in case of RAD = 0, the contract curve passes through the central point of the model (½ , ½). It will be interesting to see how the RAD affects the relative positioning of the optimal contracting curve.

Hence, we may rewrite the Pareto-optimality condition of equation [6] as:

$$(c + 2a_2\, \alpha\theta) / (c + 2b_2\, (1-\alpha)\theta \;= a_2\, \beta / \, b_2\, (1-\beta) \qquad [9]$$

To represent the central point of the model (½, ½) we shall substitute the fixed value $\beta = ½$ in [9] to get:

$$(c + 2a_2\, \alpha\theta) / (c + 2b_2\, (1-\alpha)\theta) \;= a_2 / b_2$$

The income ratio α then turns out to be:

$$\begin{aligned} \alpha &= ½ + c\,(a_1 - b_2)/4\, a_2\, b_2\, \theta \\ &= ½ + k\,(\text{RAD}/\theta), \qquad [10] \end{aligned}$$

Interestingly, α is directly expressible in terms of a θ-weighted RAD, apart from a positive constant $k = c/4\, a_2\, b_2$. It clearly affirms the previous finding that the equal income shares $\alpha = ½$ corresponds to zero RAD. It also shows that for a large value of the total expected return θ the weighted RAD/θ will converge to zero, and hence $\alpha = ½$ will tend to be a good approximation. That is, for large expected profits, the mudarabah profit-sharing ratio will tend to be an equal shares ratio.

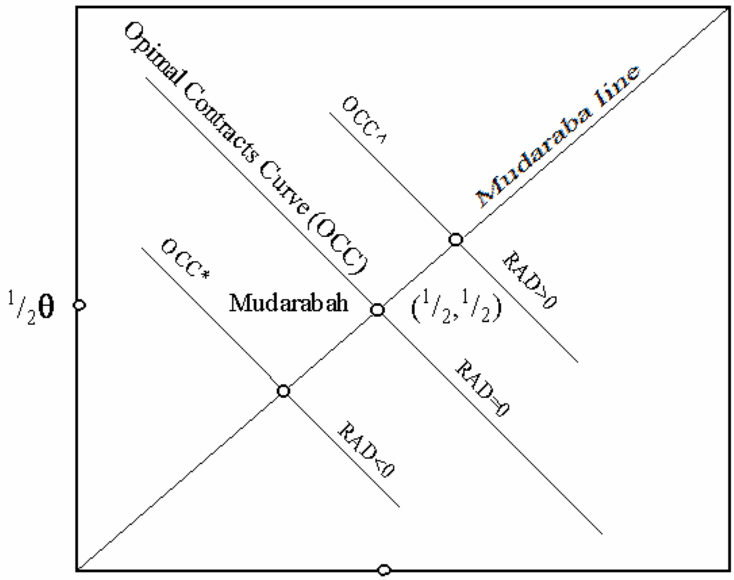

Fig. (4). The impact of RAD on mudarabah profit-sharing ratio.

Otherwise, for cases where θ is relatively small and hence RAD/θ is significantly different from zero, the relative position of the optimal contracting curve can then be shown with reference to figure (2) above. The main findings are as follows:

1. *Party A closer to risk neutrality* (RAD > 0). Here $\alpha > ½$. As expected, the optimal contracting curve will fall to the right of the point (½, ½).

2. *Party B closer to risk neutrality* (RAD < 0). Here $\alpha < ½$. Hence, the optimal contracting curve will fall to the left of (½, ½).

The figure also shows how the optimal profit-sharing ratio of mudarabah financing is affected by the RAD. For the RAD = 0, the mudarabah profit-sharing ratio is $\alpha = ½$. However, the ratio rises above ½ where RAD > 0, or drops down below ½ where RAD < 0 .

5.2 Practical implications of the break-even optimality

The fundamental implication of the above model is that income ratios are inversely related to risk-sharing ratios. This finding makes a lot sense within the assumed competitive environment where risk-averse parties behave as *expected utility maximising* agents. Bargaining for higher expected utility involves bargaining, not only for a higher income ratio, but also for a smaller risk-sharing ratio. Each party will effectively be pushing the income ratio upwards and sideways in the direction of his/her zero risk edge in as much as possible. A party's strong bargaining position is therefore simultaneously reflected in a large income ratio and a small risk-sharing ratio.

Moreover, if the bargaining power of the two parties is same, and if they adopt the same trade-off between risk and return (*i.e.* having the same attitude towards risk), the end result must a mudarabah break-even income ratio a_0 = ½ as it has been established under **RAD = 0**. Nonetheless, it is possible that the trade-off between risk and return at a_0 = ½ is not the same for the two parties *(i.e.* non-zero **RAD**). In this case one of the two parties would willingly offer his partner a partially fixed return, either in terms of a partial guarantee of interest and principal to party **A,** or in terms of a partial fixed salary to party **B**. In other words, the bargaining position a_0 = ½ is not necessarily a pure profit-sharing murabaha contract, depending on how the two parties trade-off risk and return at this point.

Alternatively, the bargaining position of the capital provider A could lead to any income ratio that is higher or lower than a_0 = ½ . Again, depending on how the two parties trade off return and risk, the agreed two-party contract could be profit-sharing *mudarabah* or any hybrid fixed return combination from the above defined set. The *mudarabah* contract is therefore just one possible break-even point on the optimal contract curve OCC that is likely to occur in the two-party model if certain conditions are satisfied. In other words, the assumed information efficient financial environment yields a potentially broad range of hybrid profit-sharing and fixed return contracts which include *mudarabah* financing as a possible case.

This theory explains why a pure equity-based model cannot prevail even under the Islamic paradigm which prohibits interest-rate financing; and why fixed returns on either side of the contract cannot be avoided. It justifies why fixed salary payments to managers or fixed return modes to capital providers have to be simultaneously accommodated even though it might be an information efficient Islamic financial market. Hence, the current efforts to avail Muslim investors with fixed return modes (*murabahah*, *ijarah etc*) seem to satisfy an imperative fact of life under the competitive market conditions. The critical question to be addressed therefore is not how to cause a radical shift from fixed return modes to pure *mudarabah* and *musharakah* financing. Rather, it is how to restrain the present upsurge of fixed return Islamic financial products towards an increasing role of equity in the present economic order.

On the other hand, the above theory has an important implication to monetary policy within an Islamic perspective. This relates to the possible use of the *mudarabah* profit-sharing ratio as a tool of monetary policy in the works of Siddiqi (1983) and Uzair (1982). Siddiqi work departs from a pure *mudarabah*-based financial environment of an Islamic economy, but this is not guaranteed even under ideal information-efficient conditions. Moreover, even if the conditions for pure mudarabah are satisfied, the maneuvering of mudarabah profit-sharing ratio would adversely affect the market conditions of an optimal appeal mudarabah profit-sharing ratio and cause a shift from pure mudarabah regime towards a hybrid regime. In other words, the financial appeal of *mudarabah* to capital providers cannot remain unchanged through interfering monetary policy with the profit-sharing ratio. Given that an Islamic economy is not necessarily a pure equity-based environment, monetary policy should utilise appropriately structured fixed-return tools on the basis of *murabahah* mark-up, ijara rates, etc in addition to *musharakah* and mudarabah tool to develop a more effective monetary policy.

Another implication of the above theory relates to the optimal mudarabah profit-sharing ratio. As it appears, this theoretical question becomes quite complicated by the unpredictable nature of an unobservable RAD. In principle, there are at least two reasons to believe that $a_0 = ½$ is the centre of gravity for the probability distribution of RAD, and any given θ under the assumed competitive conditions. First, the generation of either *positive* or *negative* RAD will depend upon the nature of the matching process of the two-party contract within the financial market. If we assume a random matching process, then RAD will be a zero expectation random variable, resulting in:

$$E(\alpha) = \tfrac{1}{2} + k\,(E(RAD)/\theta) = \tfrac{1}{2} \qquad [11]$$

In this respect, random matching will most likely neutralize the wealth effect which may account for the possibility of having different risk-aversion rates.

Second, the RAD may not differ significantly among contracting parties in the actual practice. After all, risk-aversion is an embodiment of bounded concave utility function for money. The specification of *money* utility function through the cardinal expected utility approach makes it more tenable for making sensible inter-personal assumptions than the case with consumer good utility functions. The wealth effect is often cited as a decisive factor in the inter-person comparison of risk-aversion rates. Otherwise, there are hardly any grounds for remarkable *pure taste* differences in money to account for manifestly different utility functions for money.

6. Conclusive Remarks

The main implication of the above analysis is that the risk sharing structure of a manager/financier contract cannot be viewed independently of its income sharing ratio. In particular, a broad range of financial leverages are proved to co-exist with mudarabah financing, hence a pure equity-based Islamic order is inconceivable even under ideal information efficiency conditions. An important policy implication of this finding is that the mudarabah profit-sharing ratio cannot be freely manipulated through a discretionary monetary policy without adversely affecting the appeal of mudarabah in the financial market.

The main prediction of the two-part model is that *the stronger is the party's financial bargaining position, the larger is his/her income ratio and the smaller is his/her share in risk. Conversely, the weaker is the party's financial bargaining position, the smaller is his/her income ratio and the larger is his share in risk.* Economically weaker parties are, thus, left at the great disadvantage not only in terms of smaller shares in national wealth but also in bearing larger shares in the underlying economic risk. This finding adds another dimension to the ethical adversities of an inequitable income distribution, that is the keen effort of wealthy people to shift financial risk away from their budgets.

A strong market position of capital owners is non-conducive to a pure profit-sharing Islamic system in as long as it generates a sufficiently high income ratio for capital owners, $\alpha_0 > ½$. Such high ratio is shown to induce lending at a risk-free interest, which within the interest-free Islamic system would reflect in fixed return Islamic modes that are good substitutes of interest rate lending. Apparently, whoever acquires command over capital market resources will tilt the balance of less risky returns towards his or her favour.

In particular, the pure mudarabah environment emerges as a break-even point where the opposite bargaining positions are fairly balanced. Hence, the ethical appeal of mudarabah financing relates not only to a fair risk-sharing property but also to a significant tendency towards a 'fair' income ratio of around 50%. This finding provides a more appealing interpretation to the concept of economic justice in mudarabah than the one based on pure altruistic behaviour. The prevalent trend over-emphasizes the ethical appeal of mudarabah financing in terms of its fair risk-sharing provision alone but this study extends its ethical appeal to income ratio under the assumed competitive conditions.

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APPENDIX

Proof of Equation [7].

Equation [7] establishes a linear negative relationship between the income sharing ratio α and the risk sharing ratio, which is the pivotal finding of the paper. Moving forwards from the equation [6], this is now reproduced below:

$$(c + 2a_2\theta_1)/ 2a_2 \sigma_1 = (c + 2b_2\theta_2)/ 2b_2 \sigma_2$$

Then, insert $d = a_2 = b_2$ to account for *equal risk-aversion*, the above equation can be rewritten in terms of α and β by realizing that $\theta_1 = \alpha\theta$ and $\sigma_1 = \beta\sigma$ while $\theta_2 = (1-\alpha)\theta$ and $\sigma_2 = (1-\beta)\sigma$. Hence, the above equation is reducible to:

$$(c + 2 \alpha d\theta) /2\beta d\sigma = (c + 2(1- \alpha) d\theta)/ (2 (1- \beta) d\sigma)$$

Then, through cross multiplication and algebraic manipulation, it is easy to derive the required linear relationship:

$$\alpha = m + w\beta, \quad \text{where } m = - c/2d\theta \text{ , and } w = (c + d\theta)/d\theta,$$

As regards the negativity of the relationship, this requires the additional proof that $w < 0$. This property follows from conditions [2] and [3] above. Given the restrictions at $a_1 = b_1 = c$ and $a_2 = b_2 = d$, condition [2] implies that $c + 2d\theta > 0$ while condition [3], implies that $d < 0$. Due to the last property, the condition $c + 2d\theta > 0$ implies $(c + d\theta) > 0$, and hence completes the proof of $w < 0$ as defined above.

نسبة الدخل، والاشتراك في الخطر وأمثلية المضاربة

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المستخلص. إن أدبيات التمويل الإسلامي لم تمنح العناية الكافية لتحليل أدوات التمويل اللاربوي باستخدام تحليل العائد والخطر في تحلل المحافظ المالية. هذه الورقة تستخدم الأدوات المعهودة للوصول إلى خصائص أمثلية المضاربة في إطار نموذج العلاقة التعاقدي الثنائي لنشاط اقتصادي مولد للدخل. والورقة تمثل امتدادًا لما قام به الباحث عندما وجد أن تركيبة الخطر المشترك تتناسب بشكل تام مع نسبة العائد في عقد المضاربة.

وقد توصلت الورقة الحالية إلى نتيجتين إضافيتين لهما أبعاد تطبيقية وإجرائية، وهما: الأولى وجود علاقة سلبية بين نسبة العائد وتركيبة المشاركة في الخطر باستخدام منحنى الأمثلية للعقود (OCC)، والثانية وتتمثل في أن نسبة العائد الأمثل في عقد المضاربة تتوقف بشكل كبير على سلوك الطرفين المتعاقدين حيال الخطر.