

Preliminaries to an Investigation of Reduced Product Set Finance

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Abstract. Principles of financial product synthesis from a few basic financial products constitute an interesting research topic inspired by Islamic finance. We make an effort to answer general questions that should be answered before starting to investigate the main issues concerning this topic with the formalization of financial products and principles of financial product synthesis. We also outline the outcome of some preparatory explorations, which have been conducted with the purpose to form a reasonable preliminary picture of the details of financial products that are relevant to the study of the principles of financial product synthesis.

1. Introduction

The idea of considering only financial products that are synthesized from a few basic financial products according to certain principles is both appealing from a scientific point of view and directly relevant to the practice of finance. In science, at least in computer science, it is common practice to start developing a sound understanding of complex concepts by deliberately simplifying matters like this. However, the idea is also actually used in the practice of Islamic finance. This makes it very attractive to investigate issues such as the issue whether principles of financial product synthesis can be formulated that, if applied to products that are legal according to Islamic law, lead to synthesized financial products that are also legal according to Islamic law and the dependent issue which financial products that are not legal according to Islamic law can be approximated by financial products that are synthesized according to legality-preserving principles from legal products.

We believe that before such an investigation is conducted, answers to certain general questions should be given. Among the most important general questions are the question whether the investigators can conduct the investigation impartially, the question whether the investigation can be conducted without assumptions about the validity of ethical principles, the question whether the investigation is timely, and the question whether the object of the investigation has an application perspective. We believe that, in order to achieve reliable results, the formalization of financial products and principles of financial product synthesis must form part of the investigation. We expect that methods and techniques developed in computer science will play an important part in the formalization. An interesting general question raised by this is the question what is the relevance of the investigation for computer science.

Among the more specific questions related to the formalization of financial products and principles of financial product synthesis are the question which details of financial products are needed to formulate principles of financial product synthesis and the question which details of financial products are needed to determine their legality. The more specific questions like these ones cannot be answered fully before the investigation is conducted. We have done some preparatory explorations by means of examples with the purpose to form a reasonable preliminary picture of the details referred to above.

In this paper, which was written in preparation to the investigation brought up above, we make an effort to answer the above-mentioned general questions and we outline the outcome of the preparatory explorations.

The paper is organized as follows. First, we give some classical examples of synthesized financial products and introduce the kind of finance involved (Section 2). After that, we go into the question whether issues related to Islamic finance can be investigated from a non-Islamic perspective at all (Section 3). Next, we make an effort to answer a number of general questions to which answers should be given before the investigation proposed in this paper is conducted (Section 4). Following this, we pose questions about the classical synthesized financial product examples given in Section 2 and the prevailing positions concerning their legality (Section 5). Then, we look at a loan with interest, including the particular case of a savings account with interest, with the purpose to get an impression of what it takes to formalize simple financial products (Sections 6 and 7). Thereafter, we carry out a preparatory exploration of the synthesis of a savings account with interest in the way known as *tawarruq* (Section 8). Finally, we make some concluding remarks (Section 9).

To clarify the title of this paper, we remark that we will use in this paper the term “reduced product set finance” for finance where only financial products that are synthesized from a few basic financial products according to certain principles are considered.

2. Synthesized Financial Products

In the financial literature, there are publications which go into financial products that are synthesized from a few basic financial products according to certain principles. The principles in question are compositional methods for synthesizing financial products from other financial products. In this section, we give some classical examples of synthesized financial products and introduce the kind of finance involved.

2.1 Examples of Synthesized Financial Products

From Mahmoud A. El-Gamal^[19], we take the following quotation:

Now, let’s assume that the regulators or religious authorities deemed each of transactions B and C individually to have more benefit than harm. Therefore, there were no prohibitions against either B or C. Assume finally that A equals B plus C. Then, we have a legal arbitrage opportunity that can be exploited by financially engineering A from B and C, knowing that A must be desirable to have been forbidden in the first place.

To consider a concrete example, let A, the forbidden contract, be interest-based loans, and let B and C represent spot sales and credit sales of any goods. An ancient contract called in Islamic jurisprudence *bay’u al-’ina*, or same item sale-repurchase, may thus be utilized to synthesize A from B and C. Assume that A would result in X lending one hundred dollars to Y, and receiving one hundred and ten dollars in one year. Assume that Y owns an asset S, and let B and C represent credit and spot trades in S. Now, all Y has to do is to sell S to X for a cash price of one hundred dollars, and then to buy it back for a credit price of one hundred and ten dollars payable next year. The net effect is that the asset made a round trip from Y to X and back. More importantly, the loan is synthesized since X gives Y one hundred dollars now, and Y owes X one hundred and ten dollars payable in one year. This practice is actually quite common in Malaysia, where same item sale-repurchase is not deemed forbidden, as long as the two sales are not incorporated in a single contract.

However, the majority of schools of Islamic jurisprudence forbade same item sale-repurchase without a third party intermediary. Let the third party be Z, and assume that Z is the party that owns S. One credit sale and two spot sales now need to be conducted. First, X buys S from Z and pays one hundred dollars on the spot. Second, X sells S to Y on credit, for a price of one hundred and ten dollars, payable in one year. Finally, Y sells S

to Z for the cash price of one hundred dollars. Again, the net result is that the asset S has made a round trip: X paid one hundred dollars now, Y received one hundred dollars now, and Y owes X one hundred and ten dollars payable in a year. Loan A has been synthesized again. This transaction is commonly known as *tawarruq*, which literally means turning the asset S into silver, or monetizing it.

This quotation is concerned with financial products A, B and C where an identity $A = B + C$ may hold expressing that A is synthesized by composing B and C. From the point of view of formal methods in computing this text fragment alone already leads to a range of questions.

It is not hard to find other sources about the synthesis of financial products. From Willem G. Wolters^[33] we take the following quotation (translated by us from the Dutch original):

Complex contracts emerged in which the direct imposition of interests was avoided. A well-known example was the tripartite contract also called *contractus trinus*. Instead of a loan from person A to person B for which interest was due (which was forbidden) came three contracts each permitted by the Church, which together had the same effect: a loan, a compensation and an insurance. In the first contract person A invests a certain sum of money (say 100 pounds) in a cooperation with person B (partnership contract) for the time of a year. In the second contract A sells to B a part of the profit of the cooperation, for a fixed amount (say 15 pounds), to be paid by B. In the third contract between A and B, A insures himself against the loss of the principal sum by paying to B an amount, say 5 pounds. The result is that A receives 10 pounds for a loan of 100 pounds for the duration of one year. This amounts to an interest of 10 percent per year.

In 1514 the German theologian Johannes Eck, working at the University of Ingolstadt, declared the tripartite contract legitimate.

In the preceding examples, legal assessments come into play which distinguish legitimate (*halal*) from non-legitimate (*haram*) contracts or progressions of actions involved. These examples illustrate the simplest compositions of products only. Because of their classical nature, we call them *fundamental synthesized product examples*. Such examples have a long history and are in some sense the paradigmatic examples of their kind.

The most obvious reason to seek a loan is the intention to purchase a certain good while being unable to pay the full price in advance. This decomposes in lending and buying the good with the loan. The *murabaha* contract has comparable result while avoiding the payment of interest altogether. Again we quote Wolters^[33] (taking the liberty to use the name G for the good referred to):

Person A intends to purchase a good G from B and asks the bank for a loan. The bank is willing to mediate in the transaction and asks A to promise to buy G from the bank after the bank has bought G from B. The bank is now certain that when it buys G from B it can resell G to A immediately thereafter. The bank agrees that A pays for the delivery after a certain time. For the mediation, A pays the bank a compensation. The bank subsequently buys G from B and immediately thereafter resells G to A at a higher price ('cost-plus') that must be paid after a year.

For more detailed information on financial products of this kind we refer to^[18].

2.2 Reduced Product Set Finance

We have done extensive reading on the arguments for and against Islamic finance. We draw from that reading the conclusion that the investigation that we bring up in this paper, and which we hope to carry out in the near future, will have no bearing on those arguments. The reason for this is that the point of departure of the investigation will be the perception of Islamic finance as conventional finance restricted by the prohibition of interest.

The very possibility (and fact according to some) that a significant alternative to conventional finance can be found by means of a restriction of the permissible financial methods is quite intriguing. Moreover, obtaining an advantage from imposing design space restrictions is a phenomenon known from computing. In computer architecture, the limitation of instruction sets has been a significant help for developing faster machines using RISC (Reduced Instruction Set Computing) architectures. Fast programming, as opposed to fast execution of programs, is often done by means of scripting languages which lack the expressive power of full-blown program notations. Replacing predicate logic by propositional calculus has made many formalizations decidable and for that reason implementable and the resulting computational complexity has been proved to be manageable in practice on many occasions. New banking regulations in conventional finance resulting from the financial crisis of 2008/2009 have similar characteristics. By making the financial system less expressive, it may become more stable and on the long run more effective. Indeed, it seems to be intrinsic to conventional finance that seemingly artificial restrictions are a necessity for its proper functioning. The development of these restrictions is propelled by the drastic innovations of the financial industry rather than by ethical constraints of a principled kind.

We find it appealing to systematically study forms of finance where only financial products that are synthesized from a few basic financial products

according to certain principles are considered. We coin the term “reduced product set finance” for this kind of finance. We consider the form of finance that the avoidance of interest gives rise to in Islamic finance the classical example of reduced product set finance, but recognize that Islamic finance aims at objectives not captured by reduced product set finance.

Restrictions other than the avoidance of interest have been imposed in the past. Mostly, the restrictions concerned the use of money. Among them are the restriction that only state controlled agencies are allowed to mint coins and the restriction that the issuer of coins keeps the ownership of coins — implying that holders of coins are not entitled to extract valuable materials from coins. The fact that interest rates are non-negative is a current restriction which might be removed from conventional finance. Redesigning the system without coins and banknotes using e-money only will amount to yet another restriction which is bound to be studied in depth in the near future.

Whatever the practical value of reduced product set finance, it qualifies as an intellectual challenge at least to explore its potential. Our idea is to study the most prominent reduced product set finance obtained by the prohibition of interest in a formal way. Clarification of concepts and the analysis of proper distinctions is considered prior to the assessment of economic, political and social effects of putting a particular form of reduced product set finance in place in a real economy.

3. Impartial Investigation

The question whether Islamic finance or aspects thereof can be investigated from a non-Islamic perspective at all is a complex question for which no brief and conclusive answer can be provided. However, when writing from a non-Islamic background, simply by doing so, we cannot deny to have a positive answer to that question in mind. In view of the wide range of opinions that are being voiced about the role of Islam in modern democratic countries, we believe that investigating Islamic ideas from a non-Islamic perspective requires taking up a position with regard to those opinions. Below, we explain our viewpoints on this matter. They support a context in which an impartial and systematic investigation of aspects of Islamic finance can be pursued. For none of our viewpoints we claim originality. They derive in part from^[31] and they can all be found in a diversity of sources.

3.1 Separation of Church And State

The objection that Islam fails to separate church and state and for that reason lacks modernity is flawed. The institutional separation of church and

state and the ideological separation of religion and politics are both processes which are still under development in the West. Western democracies may have been premature in declaring these issues solved. Confrontation with a substantial representation of Islam in EU countries reveals weaknesses in the maturity of these separations. Indeed this confrontation is likely to contribute to a further maturity. There is room for the assumption that Western democracies have made use of the fragmentation of Christianity to steer towards an equilibrium which has not always been convincing from a religious (in particular Christian) perspective.

3.2 Integration of Islamic Minorities

The objection, nowadays often voiced in EU countries, that Islamic minorities fail to integrate, thereby causing problems to their country of adoption, cannot be held against Islam in any way. Migration streams often lead to integration issues and this is by no means specific for migration streams from countries with Islamic majorities. There is no evidence that such minorities would be less productive in terms of integration than minorities of other origin.

3.3 Shariah and National Constitutions

The objection that Islam promotes a vision that Shariah takes preference over national constitutions, thereby being “undemocratic”, cannot be taken seriously, at least not as an objection specifically raised against Islamic thought. Christian and Judaic ideologies would prescribe similar viewpoints if significant frictions with local constitutions arise, only stated in different words. If that form of independence would at some critical stage fail to emerge, later generations of Christian communities might not praise their ancestors for their obedience to time bound constitutions.

The confidence that Christians have in democratic processes is high but limited in principle. Their acceptance of these processes as leading is connected to an implicit assumption that “Christian values are dominant”. The whole picture is turned upside down if one pretends that Christians have all seen the light of democracy and now believe that always two thirds of a population have ultimate authority on ethical matters. This infallibility of the democratic process is of course equally problematic as some similar principles that have been preached from Rome.

Indeed a strong foothold of Islam in western democracies will force Christians in those communities as much into carrying out an in depth analysis of their appreciation of democracy as their Islamic co-citizens. If their appreciation of democracy is linked to the assumption that Islam has a relatively

weak position, then the maturity issues mentioned above surface at once. Otherwise, they have either embraced democratic mechanisms as incorporating ultimate ethical values or they have recognized Islam as a “friendly force” when put to the test in extreme cases. Either way, these steps are quite non-trivial and Islam cannot be blamed for the existence of the conceptual problems which are simply endogenous to Western political thought.

3.4 Import of Islamic Thought

The import into Western Europe of Arabic and Islamic thought has been massive, but it took place long ago and non-trivial historical awareness is required to appreciate that fact. Promoting that awareness is certainly not a major objective of the current Christian-Judaic tradition. Even the conception that Western Europe mainly stands in a Christian-Judaic tradition may be contested. Several rules that underly Islamic finance have been adopted in Western finance as well. Moreover, there is no reason to assume that an Islam tradition somehow Europeanized may not flourish and have world wide impact. There is no obvious ground for a specific and lasting commitment of Western European countries to the oldest two of the three major Abrahamic religions. The mere argument that recent immigrations force several EU states into the accommodation of Islam is besides the point. Islam has never been very far away, and needless to say many of the immigrants have been invited to come.

3.5 Islamization of Modernity

The main debate between the Christian-Judaic West and Islam seems to concentrate on the interpretation and the advancement of modernity. If Islamic influence implies an inflexible commitment to an explanation of all social realities in terms of writings dating back some 1000 years and more this very lack of commitment to open-minded innovation may constitute a major cause of frictions. The well-known dichotomy between “Islamization of modernity”⁽¹⁾ and “modernization of Islam” points to this issue. If the more extreme viewpoint of “Islamization of modernity” is taken as a point of departure, which may be compatible with an EU based Islamic tradition with world-wide ambitions, then the critical debate will be about the form and objective of modernity as an ongoing and progressive process.

3.6 Competition between Christianity and Islam

If one denies the relevance of religion, it is obvious that accommodating more rather than fewer religions is the way ahead while defending some

(1) From this “Islamization of Science” and “Islamization of Finance”, subsequently resulting in Islamic finance, have been derived in the first half of the 20th century.

religions against the progress of other religions is hard to explain. If one accepts the potential relevance of religions, allowing a fair competition between them is the way to go, while supporting a cartel is not⁽²⁾. If one is convinced of the ideological superiority of Christianity or Judaism, then there cannot be an objection against the investigation of other religions and the political and economic impact of their tenets. So it seems that our work is objectionable only from a perspective that Christianity or Judaism needs protection in some Western democracies because it lacks any convincing superiority and that in addition investigating Islamic finance creates a problematic risk of infection. By doing the work presented in this paper, we implicitly state that we do not support the latter viewpoint. That may be held against us and the consequences of such objections are ours.

4. General Questions

Before the investigation proposed in this paper is conducted, answers to certain general questions should be given. Among these questions are the question whether the investigation can be conducted without assumptions about the validity of ethical principles, the question whether the investigation is timely, the question whether the object of the investigation has an application perspective, and the question whether the investigation is relevant to computer science. In this section, we make an effort to answer the above-mentioned questions. We also go into the relevance of computer science to the investigation.

4.1 Independence of Ethical Principles

The investigation will not be based on any assumption about the validity of ethical principles such as the prohibition of interest payment and charging. Independently of any such assumption, it can be observed that a very substantial community considers such forms of prohibition important. We will focus primarily on the phenomenon of interest and ways of avoiding its usage. Because no ethical objections have been published against such forms of prohibition, there seems to be no conceivable objection against taking it very seriously as a potential design decision of a modern financial system and investigating its role and meaning in technical terms. Our work can be understood in this line.

To our knowledge, a system of ethical banking based on the prohibition of interest as well as some other well-known restrictions is a subsystem of conventional banking. Investigation of the expressive power of a limited subsystem has always been considered important in computer science and the line of thought that we propose to follow is similar in spirit.

(2) Remarkably, the objection against cartels can be found in classical Islamic writings.

4.2 Timeliness

One may consider the issues to be investigated futile because the prohibition of interest constitutes a step backward from a conventional point of view. Against that viewpoint one may put forward that, given the extreme importance of interest rates in conventional finance, it is an essential thought experiment to understand in detail what happens if that mechanism is ruled out. The fact that this thought experiment has an extremely longstanding history does not imply that all of its consequences have been completely investigated. We suggest that formalization techniques that have been developed in computer science may still provide interesting and new insights in this very classical area.

So rather than asking what is lost by the prohibition of interest, one may ask what is gained by permitting interest from a situation where it is not being used. In other words, a reduced product set finance may lead to an effective system for which the addition of some feature may be less advantageous than expected by those who take the feature for granted. Besides prohibition of interest charging and payment, the prohibition of excessive and unnecessary downside risk (*gharar*), see e.g.^[17], is an equally interesting limitation of financial products, though far harder to study because of its conceptual difficulty. Indeed, one cannot require the absence of uncertainty in all circumstances and removing *gharar* seems to require a mixture of qualitative and quantitative reasoning.

For those who do not see the point of interest prohibition at all, we recommend^[26]. That paper provides a harsh survey of the practice of credit provision in the US, which suggests that at least in some cases prohibition of interest may be a plausible definite step forward. The problems inherent in a liberal market place based on credit and interest are very convincingly put forward.

Mohammad N. Siddiqi states in^[32] that more fundamental research on Islamic finance is welcome. The investigation proposed in this paper may add to such work. However, whether our points of departure will qualify as points of departure for such fundamental research in the perception of Siddiqi is another matter. Rania Kamla^[24] suggests research on Islamic accounting to move far beyond interest prohibition. His paper convincingly criticizes the position that interest prohibition is the most central distinction between Islamic finance and conventional finance. Indeed we do not suggest that an investigation of the foundations of interest avoidance and interest prohibition are necessarily high priorities in a modern research agenda on Islamic finance because many other issues may be of equal or higher importance.

Statistical work on financial techniques applied in Islamic banking as well as the geographical distribution of Islamic banking can be found in^[4,16]. The results of such work must also be taken into account when making an assessment of the timeliness of fundamental research in interest prohibition for Islamic finance at large. In the light of the above-mentioned papers, we conclude that the priority of this research is moderate at best.

4.3 Application Perspective

As outlined in Section 4.2, there seems ample justification for detailed investigations of interest free forms of reduced product set finance. However, one may still hold that interest free finance is an incoherent concept per se and that it has no application perspective. Many objections have been put forward and many papers contain thorough responses to a selection of such objections. We mention^[15] as a systematic discussion in this respect. However, there seems to be a number of questions about the rationale of interest free forms of reduced product set finance which are hardly taken into account by its proponents, to such an extent that this constitutes a risk for the application perspective of interest free forms of reduced product set finance. Here we list some of these questions⁽³⁾:

- Assume that X may borrow an amount p from Y during time interval I without paying interest. How should Y be compensated for the risk that X defaults and fails to return the amount p at the end of I ? One might say that the presence of this risk constitutes *gharar* and for that reason it need not be taken into account. That is, Y should not engage in the transaction if there are worries about X 's ability to return the principal sum. That by itself is significant, as it would point out that interest free reduced product set finance needs to incorporate additional restrictions. But the argument is weakened by the fact that the risk of default may be quite low which is an indication against *gharar*.
- The significance of prohibition of interest is far more easily understood in the context of a firm and stable metallic (or bi-metallic) standard than in a setting where money is sensitive to inflation and deflation. As it seems to be the case that classical authors all had some metallic standard in mind, or even made no distinction between money and precious metals, one may

(3) In [22] some of these arguments are taken care of and an entirely different path of argument is taken: it is argued that prohibition of interest, or rather equity based financing, is a rational mechanism which minimizes risk in some mathematically precise way. While providing grounds for interest prohibition, that argument seems not to prove the necessity of interest prohibition. So the question remains why the optimization result with regard to risk would give rise to a strict normative judgement.

question the very proposition that their viewpoints were meant to be applied to all future forms of money, including modern fiat money. In general, there is hardly enough analysis of the concept of money when prohibition of interest is proclaimed.

- Proponents of an interest free reduced product set finance often point out the disadvantages of interest payment as a means of conducting financial business. It is hard to understand why a mechanism which may sometimes, but not always, lead to adverse consequences must be entirely forbidden. If there are disadvantages correlated with the usage of interest, one needs policies to remedy these disadvantages. One cannot but conclude, so it seems, that today's proponents of interest free banking and finance base their judgement exclusively on the interpretation of classical legal scholarly writing within a variety of religious traditions. This form of analysis induces an uncompromising judgement on absolute moral grounds which is not amenable to exceptions.
- If X provides a loan p to Y , one may hold that X provides a service to Y . But equally well Y may be providing the service to X , either because Y can store p for X or because collecting interest on the loan is the best or even only method available to X to get any rewards from its possession of the money. More attention should be paid to the variety of motives for X and Y for a seemingly similar transaction when contemplating judgements of legality.
- The strongest argument against a full prohibition of interest is that the profit-loss contracts which are advocated instead are likely to have far higher transaction costs than interest compensated loans. Indeed if X , rather than borrowing p to Y , provides p to Y for combining forces in a joint enterprise that is mainly managed by Y , whereas X is entitled to some fraction of its revenues, X inevitably is dependent on being correctly informed about Y 's results. Even worse, X and Y seem to have conflicting interests (in the other sense of the word). The simplification introduced by a loan with interest, with respect to a profit-loss sharing participation in another party's enterprise, is that a complete correspondence of objectives between both parties is obtained if X is not fraudulent (otherwise both options are unsatisfactory anyhow). It follows that, in the case of a loan with interest, X can manage his part of the contract on the basis of far more abstract information about Y . The information in question may be obtained in a less intrusive fashion, which is profitable for maintaining proper relations between X and Y . Of course, this argument would disappear if the prohibition of interest was judged on a case by case basis. However, such flexibility is absent in recent writings on interest prohibition.

Many answers on these questions and combinations of them can be imagined. It cannot be claimed that these questions need to be resolved unambiguously for a research project in interest free reduced product set finance to be justified. It can hardly be denied, however, that any application of the results of research in this area will critically depend on the collection of answers that are provided to these objections to the prohibition of interest.

4.4 Relevance to Computer Science

Besides suggesting that financial product synthesis merits substantial further investigation, we also claim that doing so may be performed by means of methods and techniques which were originally developed in computer science. In principle, a proof of this claim can only be given by actually carrying out the work with the help of such methods and techniques. However, it is meaningful to speculate about the possibility that results may be fed back to the computer science setting. Below, we give some grounds on which we expect this to be a realistic possibility.

Behaviours with promises and obligations

Probably the simplest connection between computing and the world of financial product synthesis is found if one compares a contract with a control code or an instruction sequence as found in computing. The presence of a plurality of contracts between different agents suggests the presence of some form of multi-threading. Moreover, promises and obligations play an important role. Specifying the behaviour of computer-based systems in terms of promises and obligations is relatively new, however, and progress may be obtained by making use of the results of a systematic analysis of financial products.

Classification of artifacts

In computing, the classification of large collections of software artifacts in a small number of categories takes place quite often while theoretical work supporting such classifications is much less developed. A plan for reverse application into computer science is that solutions to the classification problem concerning financial product legality may shed light on classification problems which can be found in computing. Here we mention some of these latter classification issues:

- Control code (see^[9,6]) can be classified as either programmed or non-programmed (acquired by dark programming in the terminology of^[23]). More conventionally control code can be classified as either executable or non-executable. Both classifications are far from obvious, in spite of their intuitive appeal.

- Control code may also be marked as malicious. Again the intuition is clear but a formalization of this property is hard to provide.
- Computer programs are often understood as being either faulty or correct. This distinction is somehow comparable to the distinction between illegal and legal in the the case of financial products. Moreover, the connection between program correctness and program testing is difficult to assess because software testing has not been defined clearly (see^[28]).
- The role of control code in a system may be hard to assess. For example, it is difficult to classify control code as either system software or application software. It is frequently mentioned that operating systems belong to system software, but it has been pointed out in^[27] that the very classical notion of an operating system fails to have been provided with a clear definition up to now.
- In a computer-based system some of its concurrent processes may be threads (see^[8,30]). The intuition is that all threads are processes but not conversely. Which processes are threads is a matter of definition, but there is no consensus about the definitions in question. The abundant use of the terminology of multi-threading seems to stand in the way of a proper understanding of this matter.

Computer support for financial products

Computerized financial systems are among the most important applications of computers. It is reasonable to assume that an improved understanding of financial products can also be helpful for an improved understanding of computer support for such products (see e.g. the literature on Islamic credit cards^[3,29]).

Certification of compliance with ethical values

If one assumes that legal issues concerning financial products derive from the intention to design and to improve systems for ethical banking and finance, this intention by itself may be meaningful for the purpose of the design of computer-based systems. Increasingly, ethical issues appear to be central to the design of computer-based systems. It may be quite important to be able to certify that a computer-based system incorporates some ethical values that have been specified in advance. Doing so is far from trivial and making use of examples from other areas may be of help.

4.5 Relevance of Computer Science

We expect to use multi-threading as an operational paradigm for putting financial products into effect, instruction sequences as considered in program

algebra as a means for defining threads for parts of financial products whenever possible, and quantities from a meadow as a means for measuring money. Below, we give grounds on which we expect this.

Multi-Threading

Financial products, when put into effect, nearly always involve the activity of different agents. For instance, a bank is considered an agent. The different agents operate concurrently. Each agent on its own acts in a sequential fashion, usually executing a predetermined plan. Therefore, it is plausible to think of the behaviour of an individual agent as a thread. The concurrent composition of several threads needs to guarantee that each agent is able to perform the planned actions in a regular fashion, without some agents being starved due to repetitive and uninterrupted activity of other agents. Precisely this kind of parallel cooperation is captured by multi-threading for which we have developed very elementary formalizations in^[8].

Instruction Sequences

The use of instruction sequences as considered in program algebra (see^[7,30]) as a means for defining threads for parts of financial products is based on the observation that in computing an instruction sequence is a very plausible means for defining a thread. However, we do not claim that all contracts can be decomposed into a number of threads of which each is defined by means of an instruction sequence that captures the essence of a part of the contract. Indeed contracts may be too sophisticated or too involved to admit such a decomposition.

Meadows

Meadows (see^[12,13]) are appropriate mathematical structures for quantities if quantities are measured with finite accuracy, as in the case of money. The prime example of meadows is the field of rational numbers with the division operation made total by imposing that division by zero is zero. Meadows are mathematical structures to which techniques developed for abstract data types in computer science can be applied.

5. More on Synthesized Financial Products and Legality

We believe that, in order to achieve reliable results, the formalization of financial products and principles of financial product synthesis must form part of the investigation of the main issues concerning reduced product set finance. That formalization requires a clear understanding of synthesized financial products and the prevailing positions concerning their legality. In this section,

we pose questions about the fundamental synthesized product examples from Section 2.1 and the prevailing legal positions that need answers in order to gain a clear understanding. Moreover, we attempt to give a provisional answer to the question which actions or events should be distinguished when formalizing the fundamental synthesized product examples and list a number of actions which may not qualify as ethically correct from the perspective of certain legal positions.

Henceforth, in the context of financial products, the term “state” is used to refer to a snapshot of the conditions in which a financial product is when it is put into effect and the term “progression” is used to refer to a succession of actions that results from putting a financial product into effect.

5.1 Questions about Synthesized Financial Products

In spite of their respectable age of several centuries, a number of questions can still be posed about the fundamental synthesized product examples from Section 2.1. We list a number of questions that occurred to us:

- Which catalogue of legitimate basic financial products, services and contracts can be assumed for further synthesis? We mention that a list of *halal* products, with informal specifications of each of them, can be found in^[20].
- Which categories of objects need to be distinguished, and what definitions are to be used for them? Possible categories include the categories of states, agents, accounts, goods, values, contracts, binding promises^[1], non-binding promises^[14], progressions, threads^[8], and processes^[2].
- Which actions or events should be distinguished when formalizing the fundamental synthesized product examples?
- What is the semantic complexity of product legality? That is, is legality of a product based on an assessment of one of the following:
 - one or more contracts;
 - a single progression;
 - a tree of progressions that forms the behaviour of a thread;
 - a more general process behaviour;
 - a particular combination of entities of the above-mentioned kinds?
 - Which principles of financial product synthesis are used in the cases of the fundamental synthesized product examples?
- Can it be taken for granted that synthesis of products from legitimate products leads to legitimate synthesized products? If not, why can it be assumed in the fundamental synthesized product examples? In other words,

what are valid principles of synthesis? We note that special attention must be paid to the fluctuations in value of the asset or business involved between the different actions or events that make up the fundamental synthesized products.

- Is it possible to design a large space of potential products closed under certain principles of financial product synthesis that contains both legal and illegal products (according to some definition of legality), including the fundamental synthesized product examples?
- Can judgements about legality, based on some set of general assumptions, be automated in the sense that decisions about legality can be made by means of an unambiguous logic? And if so, in which formalism can the conditions that determine the legality of a contract (according to a specific legal position) best be expressed? Some candidates are progression ring notation^[11], (timed) tuplix calculus^[10], thread algebra^[8], process algebra^[2], temporal logic^[25], predicate logic^[21], and propositional logic^[21].
- Are there cases where legality of a product is not determined by its own structural properties but rather by the use that is made of it in a specific context of agents. In other words: must the relevance of the use of the product for one or more of its participating agents be taken into account when deciding about its legality?

5.2 Questions about Legal Positions

Different legal positions concerning product legality occur in practice. We list a number of questions about those legal positions that occurred to us:

- Can a comprehensive and comparative survey of the legal positions about financial products be given?
- How can a legal position be specified?
- For each legal position (given the presence of a finite number of agents, say k):
- What counts as parameter of financial products (in addition to the saver or borrower and the principal sum deposited or borrowed)?
- How many legitimate financial products exist (finitely or infinitely many, and if only finitely many: how many as a function of k)?
- Which symmetries can be found, and must roles of agents be distinguished (e.g. banks, companies, and private individuals)?
- Is it meaningful to allow auxiliary agents, and how does that impact the counting of legitimate products?

- Is there a notion of completeness for a legal position? And if so, is a finite presentation of a complete legal position necessarily equipped with rules that have infinitely many instances?
- Are there well-definedness criteria for a legal position? Examples of well-definedness criteria are that legality must depend on behaviour only and that legality must depend on externally observable behaviour only.

5.3 Further Thoughts on Legal Positions

It is an intriguing fact that according to some legal positions very substantial differences exist in the legality of seemingly related financial products or compositions thereof. Taking a semantic view from computer science as a point of departure, it must be admitted that initially no intuition about how to recognize these differences is given. The design space of potential financial products, atomic and composite, needs to be enriched with design requirements and rules in order to single out legitimate designs. Finding out about those requirements and rules must be done by asking questions to experts or, in the presence of an abundant literature, by reading professional texts about these legality judgements. The following thoughts may be important to this matter:

- If no clear requirements and rules can be formulated, machine learning can be employed to detect the patterns which are considered decisive for legality.
- A qualitative question for each legal position is whether its judgements are binary or there exists a larger range of judgements. If binary judgements cannot be achieved, the question arises how to decide about the particular cases where a binary judgment cannot be achieved.
- If clear patterns can be found, specifications of legal positions can be provided of which each comprises a sound and complete system of inference for product legality.
- Only after the fundamental composite product examples have been completely understood, one should turn attention to more complex products involving mortgages, pension funds, insurances, shares, bonds, options, futures, swaps and so on.
- An investigation of product synthesis can be helpful if certain aspects considered relevant to legality are to be better understood. Here, one may think of products which feature an excessive downside risk, products which when put into effect provide asymmetric information to those who are involved in a problematic fashion, products which when put into effect are prone to force one or more of those who are involved into undesired actions, and products which may invite usury to an excessive extent.

5.4 Actions Involved in the Use of Financial Products

The use of financial products and services involves actions (also called events), such as speech acts (promises, justifications, claims, acknowledgements, threats), transfers, sales. The following are some actions that should be distinguished when formalizing the fundamental synthesized product examples:

- X promises Y to pay a sum of money p to Y at date d for reasons r ;
- X promises Y to accept Y 's payment to him of a sum of money p at date d for reasons r ;
- X makes a payment of a sum of money p to Y via channel m at date d for reasons r ;
- X receives a payment of a sum of money p from Y via channel m at date d for reasons r ;
- X acknowledges to all agents involved in a financial product that he has received a sum of money p from Y at date d for reasons r ;
- X promises Y to buy a product or service P from Y as soon as a certain condition ϕ is met;
- X buys P at price p from Y at date d and promises to pay p at a later date e ;
- X asserts that he expects a sum of money p from Y before date d for reasons r ;
- X justifies with reasons r why he is entitled to receive a sum of money p from Y at date d ;
- X promises Y to pay a sum of money p to Y at date d if he receives a sum of money q for an insurance policy to that extent before date d ;
- X promises Y to manage a sum of money in the range p to q for Y from date d to date e , under the condition that Y transfers the sum of money in question before date d ;
- X exchanges a sum of money p with Y at date d , where p is available in different coins and/or banknotes.

We note that a promise produces a signed contract of a certain form. There are many degrees of freedom in the design of contracts. For instance, the contract is signed by X and Y or only by X , a partially or wholly unsigned version of the contract is available before it is signed or not, and descriptions of reasons are themselves included the contract or only references to these descriptions are included. Because of this, the possible promises are numerous.

5.5 Ethically Incorrect Actions

From the perspective of certain legal positions, actions do not qualify as ethically correct if they involve speculation, information asymmetry, coercion or interest. The following are examples of such actions:

- X promises Y to pay a sum of money p at date $d+2$ if it is raining in place l at date d ;
- X sells a used automobile A to Y at date d for a sum of money p , transferred by Y in cash, while X knows about technical problems with A not revealed to Y before the transaction is made;
- X asserts that he will severely damage Y 's possessions unless Y pays a sum of money p to him as a payment for a marginal or even unwanted service S ;
- X pays a sum of money $p-c$ to Y at date d with the justification that:

Y has promised X to pay a sum of money $p+i$ at a later date e if he has received a sum of money $p-c$ from X at date d or before, where c represents the costs for X of providing the loan and i represents the opportunity costs for X of lending the sum of money p during a time period of length $e-d$ to Y ;

- X receives a sum of money $p-c$ from Y at date d with the justification that:

X has promised Y to pay a sum of money $p+i$ at a later date e if he has received a sum of money $p-c$ from Y at date d or before, where c represents the costs for Y of providing the loan and i represents the opportunity costs for Y of lending the sum of money p during a time period of length $e-d$ to X .

6. Anatomy of a Noncompliant Financial Product

The objective to develop a compositional theory of financial products is a distant target. Indeed the design of non-compositional descriptions will precede successful compositional methods. Thus, a first priority for the development of a compositional theory is to obtain semantic models for very simple products like a spot sale, a credit sale, a loan without interest, and a loan with interest. Below we will focus on the last product, as the most prominent example of a financial product.

From the perspective of a given legal position, compliance of a financial product can be assessed. Some very well-known financial products are considered non-compliant from the perspective of some very well-known legal positions, in particular the classical legal positions which came about in a stable

form from the internal proceedings of the mainstream Abrahamic religions mainly prior to the reformation of Christianity.

However clear this state of affairs may be at first sight, from the perspective of formalization, the question must be posed how much information regarding some financial product must be provided to generate a judgement of non-compliance in a reliable way. This question, which can be posed about all potentially non-compliant financial products, including the most elementary ones, is about definitions. The issue is how financial products such as a loan with interest must be formally defined. In the remainder of this section, we look at a loan with interest with the purpose to get a preliminary picture of how it must be defined. We are looking for a definition in the strict sense of a definition that provides a semantic model⁽⁴⁾.

6.1 A First Definition of a Loan with Interest

One might say that, if the legal position is “Islamic finance”, without further specification of some specific direction of thought, and the financial product is defined as “taking a loan that carries interest, including the action of interest payment”, we have an example of non-compliance. But this is not the insight that we are after. The problem lies in the far to great dependence on the meaning of the terms loan and interest.

6.2 Splitting a Loan with Interest into Two Transactions

More detail is given in the following description. A loan with interest comprises the following elements:

- A borrower X and a lender Y , both assumed to be persons for simplicity. Y offers a service to X by enabling X to use a quantity of Y 's money, called a loan, during a certain time period.
- A principal sum p , which is a quantity of money.
- Dates d and e , with d before e , such that on date d a sum of money $p-c$ is transferred from Y to X and on date e a sum of money $p+i+c'$ is transferred from X to Y . Here, i represents the opportunity costs for Y of lending the sum of money p during the time period $e-d$, c represents the costs for Y of providing the loan, and c' represents additional costs for Y .

A definite weakness of this description of a loan with interest lies in the unclarity about the relation between the two transfers. Is the relation a causation, a relation of justification, a relation of enabling or a relation via a

(4) The meta-theory of definitions as given in [5] will implicitly be used.

unique third object to which both transfers refer, such as a signed contract whose preparation precedes the two transfers?

6.3 A Purely Contractual Definition of a Loan with Interest

In a purely contractual view, the borrower and the lender both sign a single contract in which they promise to make the two transfers with appropriate references to the contract. Implicitly that contract contains the concept of interest payment, though no transfers are involved in a contractual definition.

The difficulty with a viewpoint where the legality of contracts is assessed instead of the legality of progressions is the fact that progressions do not follow unambiguously from contracts. If several related legal contracts are partially honoured, it cannot be excluded that the resulting progression coincides with the most plausible progression that may arise when honouring an illegal contract.

6.4 Aspects of a Semantic Model of a Loan with Interest

A semantic model of a loan with interest has to cover the following aspects:

- at ground level, the state of a loan with interest is made up of agents, accounts, ownerships of accounts by agents, sums of money on accounts, and perhaps goods and ownerships of goods by agents;
- on top of the ground level, there is a contract level at which there is a collection of contracts at certain stages in their life-cycle;
- on top of the contract level, there is a plan level at which there is a plan of future behaviour for agents based on contracts;
- on top of the plan level, there is a history level at which there is a selection of historic information about preceding states at the other three levels;
- transitions from one state of a loan with interest to another take place as a result of actions performed by agents.

There are links between elements from the different levels that need to be maintained, e.g. the links between planned actions and the contracts that give occasion to them. The following marginal notes with regard to the plan level are in order. If several agents are involved in future behaviour, their joint future behaviour is a matter of multi-threading. The plan of future behaviour may change from state to state.

The above-mentioned aspects are by no means specific for a loan with interest. However, we claim that a semantic model already has to cover them in the case where the financial product is a loan with interest.

The semantic model of a loan with interest explicitly or implicitly includes the relevant sequence or sequences of states (sequences if putting the loan with interest into effect can be done in several ways).

Given the above-mentioned aspects, it is convenient to look upon a loan with interest as a transition system or an abstraction thereof. This allows for a process, a thread or a progression, each constituting simplified and abstract forms of transition systems. The states of the transition system consist of four components, one for each of the above-mentioned levels. Appropriate abstractions can be given in terms of equivalence relations on transition systems. Because the states are not fully abstracted from in the abstractions in question, these equivalence relations are reminiscent of the one found in process algebra with signals^[2], where signals play the role of states.

If the transition system is cycle free, histories up to some state can be reconstructed unambiguously, which makes the fourth level redundant. However, with all transition systems but the smallest ones, it is probably significant to make use of a history level and to do away with the notion of a history as formed by a path from the root through the transition system.

6.5 A General Definition of Interest Is Elusive

In spite of the progress made above, the very concept of interest has many facets and a single definition that covers all instances as a special case seems to be absent. This state of affairs is very well-known in the theory of computing. For instance the subject of parallel computing leads to many different points of view, notations and semantic models of parallel computation, with no model being the ultimate most general one. The difficulty of providing a general abstract conception of interest, say paid by X to Y , comes from the many degrees of freedom, each of which requires a choice to be made for. Among these degrees of freedom are:

- X provides a service to Y or conversely;
- X is stronger in terms of wealth and power than Y or conversely;
- X takes the initiative or Y does so;
- X is in need of money or Y has an excess of money;
- X pays interest to Y at the same time as he repays the principal sum to Y or X pays interest to Y periodically;
- X knows the risk that he defaults and fails to repay the principal sum to Y or X does not know this risk;
- Y knows the risk that X defaults and fails to repay the principal sum to him or Y does not know this risk;

- Y splits the risk that X fails to repay the principal sum from the risk that X fails to pay interest or he does not split these risks;
- there exist different ways in which Y can cover the risks;
- there exist different ways in which Y can optimize the probability that X will be able to repay the principal sum to him.

It may be concluded that a definition of interest has only a chance with a fully specified example of a loan with interest instead of working with an abstract example.

6.6 Borrower in Need of Credit

We consider the case where X borrows a sum of money p from Y and X needs the sum of money. Although this seems to be an obvious case of loan with interest, there seem to be significant problems:

- Y will not lend p to X unless Y has found evidence that X is able and willing to borrow p from Y .
- X and Y must agree on some contract which both have acquired from a third party Z , a broker, who provides example contracts and mediates in a deal.
- X takes the initiative by looking into the portfolio of contracts which Z has on offer and selecting a contract. Then X signs a contract, say C , which he considers applicable, sends it to Z , who approaches Y and finds out if Y is willing to sign C as well. If Y is not willing to sign C , this is communicated by Z to X and there is no deal on the basis of C , after which X may either look for a different contract from Z 's portfolio, a different lender or a different broker, or give up on obtaining a loan altogether. If Y signs C as well, a signed copy is returned to Z who (i) sends a copy to X , (ii) informs Y about time, method and amount of payment to X , and (iii) informs X about the time of payment, so that X can warn Z if no payment has been received by that time. If X has to warn Z , Z will make a claim to Y on behalf of X and there is no deal. If X receives the money in time, Z prepares to inform X later about time, method and amount of repayment.
- The contract C will specify how Y will obtain guarantees that p will be repaid. There are several possibilities:
 - X may hand over to Y some valuable good which has a market price above p . X permits Y to sell these valuable goods if the money is not paid back on time. Y is not permitted to use the valuable goods for any other purpose.

- X may possess some valuable goods which are handed over to Y once X fails to return the principal sum on time. In this case, the difficulty is that Y must in addition see to it that X will not sell these goods earlier, leaving Y without guarantees after all.
- X may have an income which can be inspected by Y and which Y , as the need arises, can claim a share from until debts have been repaid. In this case, it is a complication that the income may be lost before the end of the planned period of the loan.

Each of these possibilities is different and broker Z must offer different contracts for each of them. It seems that the first possibility is the simplest. However, there is the complication that X needs to obtain guarantees that Y will return the valuable goods upon X 's repayment of the loan. The contractual side of such guarantees is complex. If X does not insist on such guarantees, it is far simpler for X to sell the valuable goods instead of taking a loan.

7. The Example of a Savings Account with Interest

Even the simplest example of a borrower in need of a loan as described in Section 6.6 is quite complex. The complexities in question may even defeat credible attempts to formalization. For that reason, we will now consider a simpler example where the lender has an excess of money.

7.1 Outline of a Savings Account with Interest

We suppose that person X has obtained a gift or inheritance consisting of valuable goods which he can sell on a market. Because X has no need of the goods and storing them is a burdensome task, he sells the goods for a sum of money p . Now X must store p until he wants to make use of it. Doing so himself creates a risk of theft or loss due to misfortune, and for that reason X intends to find a bank Y who will take p as a loan and repay this sum after an agreed time. Because p is not too high, the ability of Y to repay this principal sum is guaranteed by the state and X does not need to worry about that problem.

Y offers to repay $p-c+i$ instead of p , where c are low transaction costs independent of p and i is defined as a fraction of p , with the argument that (i) it can apply p to a portfolio of useful projects which on average produce significant returns on investment, (ii) it considers it fair to allow X some share in that profit, (iii) a competing bank, say Y' , offers to repay $p-c+j$ where $j>i$, but Y claims that this promise may induce a risk that Y' fails and that the subsequent process X needs to follow to make use of state guarantees is quite unattractive, (iv) the fractional-reserve banking system allows it to produce loans to other clients in need of money on the basis of the fact that it has

obtained the deposit p , and in this way depositing p may be considered a contribution of X to the local community which merits a reward.

The borrower (bank) thus offers a compensation for money put on deposit. In economic terms, Y is the stronger agent and in addition to this he has the backing of the state to guarantee his obligations towards X . In this case, it is quite hard to see why it should be forbidden that Y repays $p-c+i$. In this scenario, it is quite plausible that X has a surplus of money available over the sum p he intends to deposit. We will make use of that fact when considering the options for a synthesis of this loan from permissible products.

7.2 Details for a Savings Account with Interest

The case of a lender X who has a surplus of money for which he asks a borrower Y to keep it in storage requires many details for its formalization: (i) a broker Z should produce model contracts from which X can make a selection, (ii) Z mediates the communication between X and Y leading to a contract signed by both X and Y with copies available to X , Y and Z , and (iii) Z can apply for law enforcement if Y fails to repay the principal sum plus an agreed additional sum.

The model contract which X obtains from Z after having made a selection should be a model in which the principal sum p can vary over some significant interval. By providing a formula $p-c+q \cdot p$, where c and q are constants, to determine what has to be paid back by Y to X after a given time period t , a causal relation between p and i is imposed.

The repayment must be firmly linked to the original transfer of the principal sum p . How that link is formalized depends on the formalism. For instance, one may employ a system wide unique indexing of events by means of a progressive sequence number which can be used to refer to preceding events.

A savings account with interest is a financial product which is considered entirely unproblematic and even virtuous in Western finance. That judgement is not shared by Islamic jurists who claim the presence of prohibited *riba*⁽⁵⁾.

7.3 Justification of the Interest Presence Claim

One may criticize the example for being marginal in terms of interest payment. Indeed it seems to incorporate few of the aspects which have historically led to a condemnation and subsequent prohibition of the payment of interests. The importance of savings accounts with interest lies in the role that

(5) We will use *riba* to stand for interest, though this identification is by no means unchallenged.

they fulfil in Western finance whereas they demonstrably contradict the prohibition of *riba*. The justification of the example is as follows:

- The risk that X will not be repaid is minimal. Stronger guarantees of repayment cannot be imagined. It is likely that the probability of restitution failure is lower than the probability of theft or loss if the sum is kept by X in his own physical possession.
- The reward $-c+i$ splits into two parts. The transaction costs c are constant and non-objectionable. The increment $i=q \cdot p$ is obtained by X without performing any additional work or investment, or taking any noticeable risks. Because $-c+i > 0$, the increment i is significant.
- X acts out of free will, X is aware of all details of the entire transaction, X incurs no significant downside risk, X may freely choose a service provider from a number of offerings mediated by Z . Thus, no other problems exist with the transaction.
- It may be concluded that the transaction is unproblematic except for its incorporation of the increment $i=q \cdot p$. The qualification of that increment as interest is justified on the basis of the preceding three points and the fact that a causal relation between p and i is imposed (see Section 7.2).

7.4 Is the Example Pivotal?

We contemplate the virtue of a systematic investigation of possible reconstructions of a savings account with interest by synthesis from a collection of more primitive permissible financial products. The example of a savings account with interest is pivotal in the sense that it seems to be the simplest conceivable example that admits a complete and concise description. At the same time it is quite realistic. However, if Islamic jurists would consider a savings account with interest *halal* after due consideration, the merits of disproving that it can be synthesized from a collection of other and more primitive *halal* products disappear, at least from a point of view of financial system design. Assuming, however, that all forms of interest are *haram*, a viewpoint that can be found in many sources, the relevance of a savings account with interest for our objectives becomes very significant in view of its simplicity in comparison to other products which provide credit for a weaker party in need of financial support.

8. Preparatory Exploration

Taking a savings account with interest as the central example of a prohibited financial product, one faces the questions (i) how to design a portfolio of permissible basic products, (ii) how to specify which compositions of products

are permissible, (iii) how to prove that a savings account with interest cannot be synthesized by means of permissible methods of composition from permissible basic products.

At this point, it needs to be asked to what extent a consistent design of a reduced product set financial system must comply with the requirement that a savings account with interest cannot be synthesized from its basic products. Intuitively, this seems to be a compelling requirement. And the fact that provable non-synthesizability for a specific target product is a meaningful design objective definitely suggests that techniques from formal methods in computing may be needed to analyze compliance with that objective.

In the remainder of this section, we carry out a preparatory exploration of the possibility to synthesize a savings account with interest in the way known as *tawarruq*.

8.1 On Interest Free Reduced Product Set Finance

If a savings account with interest can be synthesized from basic products in some financial system, this implies that, in functional terms, what a savings account with interest provides cannot be forbidden. Rather it is the way in which a savings account with interest is described which is disapproved. As it stands the issue is open as far as we know, and it may be formulated as a technical question as follows:

Are designs of reduced product set financial systems which guarantee provable non-synthesizability for a savings account with interest possible, or is one facing the situation that the functionality of a savings account with interest must be considered unproblematic, because its synthesis from basic products cannot be prevented in a plausible way, whereas the particular form of the description of a savings account with interest is considered problematic?

The two possibilities mentioned above are very different and the understanding of the principles of prohibition of interest depends critically on resolving this issue. From the point of view of development of theory of financial systems, it seems preferable if one or more designs of reduced product set financial systems can be established which guarantee provable non-synthesizability for a savings account with interest — viewed as a functionality. If that is not possible, an interpretation where the description of a savings account with interest is simply used as a notational shorthand for an expanded and functionally equivalent synthesized product can hardly be avoided.

8.2 The Progression Architecture to Comply with

In order to prove anything about the possibility to synthesize a savings account with interest from more primitive products, a limitation of those primitive products is needed. The more general form of an impossibility theorem one aims at, the harder it becomes to map out the preliminaries and to state and prove the result. The simplest way to go ahead is to consider known ways to synthesize a savings account with interest from other primitive products. In the beginning of the paper, we have sketched three such scenarios: *tawarruq*, *contractus trinus* and *murabaha*. Of these *tawarruq* and *contractus trinus* synthesize a purely financial product, whereas *murabaha* is a sales transaction involving a non-monetary good. *Contractus trinus* involves an insurance policy, which adds complications that we prefer to avoid. This leaves us with *tawarruq* as a most relevant scenario at this stage. A synthesis of a savings account with interest by means of *tawarruq* involves the following progression π of actions when put into effect:

1. X asks Z to prepare a portion G of a certain good that has exactly the price p ;
2. X buys G at price p from Z ;
3. X sells G at price $p-c+i$ to Y where it is agreed that payment is due at time $\text{now}+t$;
4. Y sells G at price $p-\frac{1}{2}c$ to Z .

At this level of abstraction, the progression π cannot itself be considered to constitute a synthesis of a savings account with interest from permissible primitive products. Rather it constitutes the externally visible behaviour of such a synthesis, provided the synthesis is successful and also under a number of conditions that will be uncovered only when attempting to synthesize a savings account with interest.

We say that π is a *progression architecture* for the synthesized product under investigation. It represents the requirement that at some level of abstraction the given actions take place in the given order when putting the synthesized product into effect. Whether such a product actually exists is immaterial at this stage.

The plan is to take the progression architecture π as a point of departure and to study the ways to synthesize a savings account with interest that comply with π or a plausible variation of π . Putting a product into effect may involve many intermediate steps.

Regarding the the progression architecture π , the following assumptions are made:

- It is certain that the value of G does not significantly degrade in the time span t . It is also certain that the value of G does not change in the much shorter time spans between the four actions of π . If G is a portion of a precious metal, it may have this property; but if it is a portion of a non-durable good, it will not.
- Z is able to prepare a portion G of the good at the countervalue of a sum of money p (within a range that may be implicit in the original and subsequent detailed descriptions of a savings account with interest). Here it may be of importance that X has been able to prepare a sum of money p that consists of coins and banknotes, which restricts the range of values that p may take.
- The good itself is not money. Otherwise interest is involved in the third step of the progression architecture π . This means that a philosophical position concerning what constitutes money is needed (see for instance [5] for an attempt to survey this matter). Anyhow, the good cannot be used as a measure of value and the values that a portion can take are less fine grained than monetary values. For instance, G may be delivered in fixed size blocks of gold and Z may refuse to partition the blocks.

8.3 Refinements of the Progression Architecture

Because X intends to save, it may be assumed that more money is available to X and that X can buy a more expensive portion of the good when needed. This is relevant in the light of the assumption that the values that a portion of the good can take are less fine grained than monetary values. It leads to a variation π' of π which takes care of the fact that X is not exchanging money for money with Z :

1. X requests Z to prepare a portion G of a certain good that has a price p' , where $p < p'$ and p' is chosen by Z as low as possible such as to be able to deliver G within cost $\frac{1}{2}c$;
2. X buys G at price p' from Z ;
3. X sells G at price $p' - c + i$ to Y where it is agreed that payment of $p' - p$ is due at once, i.e. now, and payment of $p - c + i$ is due at time now + t ;
4. Y sells G at price $p' - \frac{1}{2}c$ to Z .

The real complication that must be dealt with lies in the flow of contracts that must underly this transaction. Contracts provide participants with assurances that guarantee that their actions make sense or, in other words, that their actions will indeed emerge in the progression that results from multi-threading the progressions of their individual plans. In particular:

- Z needs to be sure that X will buy G at price p' before he prepares G ;
- X needs to be sure that Y will buy G at price $p'-c+i$ by means of a credit transaction before he buys G from Z ;
- Y needs to be sure that Z will buy G back at price $p'-\frac{1}{2}c$ by means of a spot transaction.

At this stage the progression architecture π' must be refined in order to take into account the signing of contracts. At least three contracts are needed and the contracts involve conditional obligations. This leads to π'' :

1. X and Z sign a contract C_1 (where Z takes the initiative) that, if Z prepares a portion G of a certain good as specified in π' , X will subsequently buy it from Z at the price specified in π' ;
2. X and Y sign a contract C_2 (where X takes the initiative), with reference to contract C_1 , that, after X has bought G from Z , Y will buy it from X by means of a credit transaction as specified in π' ;
3. Y and Z sign a contract C_3 (where Y takes the initiative), with reference to contract C_2 , that, after Y has bought G from X , Z will buy it from Y by means of a spot transaction as specified in π' (under the assumption that earlier X has bought G from Z in accordance with contract C_1);
4. X requests Z to prepare a portion G of a certain good that has a price p' , where $p < p'$ and p' is chosen by Z as low as possible such as to be able to deliver G within cost $\frac{1}{2}c$;
5. Z informs X , with reference to contract C_1 , that he has prepared G ;
6. X buys G at price p' from Z ;
7. X informs Y , with reference to contract C_2 , that he has bought G ;
8. X sells G at price $p'-c+i$ to Y where it is agreed that payment of $p'-p$ is due at once, i.e. now, and payment of $p'-c+i$ is due at time $\text{now}+t$;
9. Y informs Z , with reference to contract C_3 , that he has bought G ;
10. Y sells G at price $p'-\frac{1}{2}c$ to Z .

Having progression architecture π'' for a synthesis of a savings account with interest at hand, we may start looking for positive as well as negative results concerning the possibility to synthesize a savings account with interest. Positive results depend on conditions concerning the readiness of Z and Y to sign contracts C_1 and C_2 , respectively, when requested by X to do so. Readiness requires that all contracts are prepared before any of them is signed and that they are signed in a plausible order. Covering the preparation of the contracts and providing a plausible order in which the contracts must be signed leads to π''' :

1. X and Z prepare a contract C_1 (where Z takes the initiative) that, if Z prepares a portion G of a certain good as specified in π' , X will subsequently buy it from Z at the price specified in π' ;
2. X and Y prepare a contract C_2 (where X takes the initiative), with reference to contract C_1 , that, after X has bought G from Z , Y will buy it from X by means of a credit transaction as specified in π' ;
3. Y and Z prepare a contract C_3 (where Y takes the initiative), with reference to contract C_2 , that, after Y has bought G from X , Z will buy it from Y by means of a spot transaction as specified in π' (under the assumption that earlier X has bought G from Z in accordance with contract C_1);
4. Y and Z sign contract C_3 ;
5. X and Y sign a contract C_2 ;
6. X and Z sign a contract C_1 ;
7. The remaining steps of π'' are steps 4 to 10 of π'' .

8.4 Plan of Technical Work

On the basis of the presented preparatory exploration, it should be possible to derive formal results concerning the existence and non-existence of synthesized reconstructions of a savings account with interest. General conditions will determine about the matters that have to do with willingness of involved parties to sign contracts, certainty that signed contracts are obeyed, fluctuations of the value of the good during different steps of a progression, knowledge about such fluctuations to parties involved, and the legality of ignoring such knowledge. In order to make progress from this stage onwards, attention must be paid to the development of a portfolio of permissible financial products at least involving a spot sale, preparing and signing certain contracts, and informing certain parties.

It seems plausible that at least one credit sale transaction is needed for any synthesis of a savings account with interest. However, proving this still constitutes a challenge because it requires a complete and plausible set of assumptions on available products and compositions thereof.

A positive result is likely to be found if packaging of the contracts in a single contract with three signatures is permissible. This is a deviation of π'' which removes the need to be able to cope with the preparation of contracts. Still there is a non-trivial problem about dealing with the consequences of fluctuations of the value of G : X can only expect that an overall contract C is signed by Y and Z if the fluctuations are considered unproblematic by them. If fluctuations may become large, there emerges a risk (*gharar*) which Y and Z may not be entitled to ignore at their own responsibility.

Another complication with the single contract case is connected with the status of the good of which G is a portion. If the value of G is completely stable, which is very helpful for getting the contract(s) signed, then the shaping of the good into units of a fixed form and weight that have a higher value than a unit of money may not suffice to demonstrate that the good itself is not money. A theory of non-monetary goods is needed in this case to substantiate the non-circularity of the synthesis of a savings account with interest. A thorough analysis of the single contract case must be performed before any significant progress on the multiple contract case can be made.

Of course, one may simply rule out the single contract case by disallowing it in an ad hoc fashion because it seems too close to a savings account with interest. However, in that case the intriguing fact may be missed out that deriving the prohibition of a single contract deal from a prohibition of a savings account with interest depends on implicit assumptions about the possibility to use the good involved as a measure of value and the fluctuations of the value of portions of that good. Those assumptions should be made explicit before embarking on introducing a rather unsystematic additional prohibition.

9. Concluding Remarks

We have made an extensive preparation to an investigation of issues concerning the form of finance that the avoidance of interest gives rise to in Islamic finance. This form of finance is the classical example of the kind of finance where only financial products that are synthesized from a few basic financial products according to certain principles are considered. We have coined the term “reduced product set finance” for this kind of finance.

Firstly, we have made an effort to answer the general questions that, in our opinion, should be answered before the investigation is conducted from a non-Islamic perspective. Secondly, we have listed more specific questions about classical synthesized financial product examples and the prevailing positions concerning their legality that need answers in order to gain a clear understanding of synthesized financial products. Thirdly, we have done a preparatory exploration of the synthesis of a savings account with interest in the way known as *tawarruq* to form a reasonable preliminary picture.

We conclude that the investigation is feasible, but that its outcome is entirely open. It is plausible that techniques from formal methods in computing can be applied successfully. We claim that in the long run results may be fed back to computer science, which may constitute a justification of the work in itself.

The merit of this paper may be doubted in the absence of solutions to the main problems raised by it. Computer scientists often produce solutions to problems for which the relevance must be analysed or even discovered

afterwards. However, there is the principle of though that, in some cases, it is not worth to solve a problem if providing a solution of the problem is needed as a justification to state it. This principle is nowadays seldom applied in computer science. In the case of the problems that this paper is concerned with, we have made the decision that solving the problems is only worthwhile if they can be stated convincingly in advance. The methodological difficulties involved in solving the problems must be analysed and contemplated beforehand. Their significance should not be made dependent on their answers. Ideally, the problems should be stated such that the task of stating the problems and the task of providing answers may be performed by different people or teams.

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