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Profit-Loss Sharing Contract Formation Under Zero Interest Financial System

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Abstract. For financing businesses under a Zero Interest Financial System the primary desired mode is Profit Loss Sharing (PLS) contracts. However, the reality is different. An alternative, favored and dominant alternative is Mark-up (MU) financing. Lately, MU's risk-sharing features and its rate determination process have come under scrutiny. Moral hazard, tax evasion, duration of financing contracts, etc., are cited as reasons for not using PLS. Perhaps failure to craft a sound PLS contract has forced a dependence on MU. Using the profit maximizing microeconomic model of a firm, we investigate a risk-neutral entrepreneur's willingness to make a PLS contract under two different situations – first, when PLS is the only contract available and second, when both MU and PLS are available. We were able to determine not only the share rate, but how financing duration, the MU rate, risks borne by the parties, their comparative market power, negotiating aptitude and transparency affect it.

Keywords: Mark-up, Murābaḥah, Profit-Loss Sharing, Mushārakah, Zero Interest Financial System, ZIFS

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1. Introduction

Profit and Loss Sharing (PLS) and Mark-up (MU) are the two main modes of financing under a Zero Interest Financial System (ZIFS). Out of these, PLS is where profits are shared in a pre-agreed ratio while losses are borne in proportion to equity participation, and these types of contracts are widely accepted to be the most desirable under Islamic banking (Siddiqi, 1988; Khan, 1992; Mirakhor, 1987). Mark-up (MU[1]) is a mode of financing, where an existing tangible asset is initially purchased by the bank at the request of the credit-seeking buyer and then resold to the buyer on a deferred sale basis at a higher price that includes cost and a profit margin (mark-up), both known to the two parties. Whenever MU is determined in relation to an interest rate index such as LIBOR (London Inter-Bank Offered Rate) or US short-run Treasury bills rate, MU contracts may open a back door to interest[2]. So, while MU is permissible, some have argued it should be restricted or avoided (Zaher and Hassan, 2001). Chapra (1985), and Kahf and Khan (1992) are of the opinion that MU is more likely to violate underlying Islamic religious rules.

The ZIFS literature suggests that as credit business grew globally, MU began to dominate PLS in financing investments despite the prospect of higher profitability of PLS. That is precisely what economists promoting ZIFS were not expecting. Although risk-sharing by banks was argued to be fair, progressive, and at the same time socially and economically stabilizing, it appears that either from the demand side or the supply side, or both, PLS financing is having a hard time catching on (Khan, 1995). Ahmed (2002) while referring to the above cited points adds that how banks play their role as financial intermediaries may be instrumental in the expansion of PLS.

Various reasons have been cited why PLS financing has such a small market share. The main reason for the dominance of MU is its consistency with the current interest-based financial system (Khan,

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(1) In Islamic finance literature, this is usually called *bayʿ al-murābaḥah*, or *murābaḥah* for short.

(2) To note, however, for an optimization scheme to be sound some form of benchmark opportunity cost needs to be included for all the resources being utilized. Now, instead of using LIBOR from interest-based economies, one may conjecture alternative opportunity cost index from within ZIFS by using weighted indexes based on market wide return and market share on MU and PLS investments.

1995). MU is the closest Islamic substitute for interest. The deterministic MU financing is easier to contract and it facilitates predictable cash flows and bookkeeping. MU also makes sense in cases where no tangible profit stream exists (Homoud, 1974; Ismail, 1989). Also, to the extent a ZIFS-based economy is short-term trade-driven (not requiring long-term, riskier production-orientation), MU financing is the only logical financing mode. Khan (1995) provides information for a number of short-term trade financing Islamic banks where MU is dominant. Among other reasons for the preference of MU are moral hazard (MH) on the part of the borrower (Khan, 1983; Tag El-din, 1991; Siddiqi, 1988, 1993; al Qari, 1993) as well as their desire to evade taxes (Aburime and Alio, 2009) by under-reporting profits, the failure to provide initial equity to mitigate borrowers' incentive to exhibit MH (Chapra, 1985). This ignores the fact that providing initial equity forestalls the effect of adverse selection (AS) through encouraging self-selection by the firms, the non-diminishing nature of PLS repayments imposed by lending banks (Hassan, 1992), and the inability of the firm to retain and reinvest undistributed profit (Khan, 1995). Finally, owing to risk aversion, the suboptimal capitalization of banks (80% of 79 reporting; Iqbal et al, 1998) has made portfolio diversification between MU and PLS very challenging.

However, there exist some other rarely mentioned and clearly unexplored policy issues that push PLS lending down to its current level. MU financing was initially designed for cases where prior possession of the goods is possible. PLS financing was supposed to take care of cases where there is an identifiable flow of future profit streams from the project. In contrast, MU has been allowed to be extended to the for-profit sector by parsing such businesses into two components: MU financeable where banks can take possession of the goods and MU non-financeable where the possibility of taking possession is not an option because the items excluded from being funded are service products and not goods. MU financeable is then further extended to include a sale contract between the seller and the buyer of an asset transacted before it comes into existence (*istiṣnāʿ*). Consequently, MU has been allocated a much larger market access than one would expect under the concept of permissible trade (Qur'ān-2:275; Asad, 1980). By extending the horizon of the prospect for MU, most of the financing needs now-a-days can be satisfied through MU while PLS is becoming increasingly obsolete.

There is another Qur'ānic edict that is being ignored. While borrowers are given substantial leeway in drawing up the financing contract (Qur'ān-2:282; Asad, 1980), in today's ZIFS they seem to be restricted against exercising that right by the latitude enjoyed by the banks. The firms simply fill out a standard boilerplate form that rises to become the contract. Thus, PLS financing contracts may be borrower unfriendly and may be driven by the political economy favoring big money even in supposedly equity-driven ZIFS economies.

Interestingly, the lack of PLS financing can be a nascent market problem impacting it in two distinct ways. On the one hand, the spread of riskier PLS financing may be impacted by the presence of a threshold capitalization level that a ZIFS bank must attain before undertaking it. So, smaller banks as well as newer banks that are attempting to stabilize their cash flow and guarantee some level of respectable expected return to their depositors may limit their PLS portfolios. On the other hand, there is bilateral market concentration – i.e., there are a handful of such banks and the borrowers are few and large (e.g., the government) with a willingness to accept the convenience of the extended reach of MU-type financing. To that extent the market is skewed against PLS. Owing to the profitability of expanded demand and increased size of MU finance, ZIFS banks may not only ignore smaller borrowers, they may also raise the MU rate. Khaled and Khandker (2014) point out an optimization rule whereby a ZIFS bank allocates funds between MU and PLS alternatives. A possibility exists that the underlying factors of that marginal rule limits how much financial capital is deployed to the PLS sector.

Finally, while the ZIFS may be normative by definition, questions arise as to whether it is properly structured to be normative by execution. The latitude given to MU-type financing suggests leniency in normative judgments. In Islamic social, economic, moral and ethical structures (in reference to intoxicants, gambling and prostitution, and interest-based lending), we note an underlying current of modern socio-economic management. While the cost of supply has been made severely high, the preference for them has been similarly discouraged, thereby eliminating the entire market for the product. Paradoxically, under modern ZIFS economies, to the extent MU-type financing does not anticipate this structural phenomenon by the ease of expansion that it has been afforded, it causes the normative definition to be at cross-purposes with the design promoting it.

In an effort to boost acceptance and popularity of PLS financing, some authors like Khan (1995) make moral pleas to Islamic banks to take greater responsibility of sharing the risk of PLS projects. When ZIFS are already defined in the light of normative economics, further moral pleas may have limited success in promoting PLS financing in a free market profit-maximizing banking system. We contend that, given regulations or socially sanctioned parameters of organizations and operations, profit-maximizing banks will undertake PLS financing only when their risk adjusted expected cumulative profit from PLS financing will be higher than that of MU financing. It is to this end that our research is directed. We believe that if the nature of PLS contracts are properly enunciated in a technically sound manner, and the risks and rewards of their undertakings are clearly specified, it will remove some of the uncertainty surrounding PLS financing contracts. Self-interest will automatically lead a profit-maximizing bank to start sharing the risk of projects through PLS whenever circumstances prevail. Khaled and Khandker (2014) are among the first to look through the lens of profit-maximization to analyze ZIFS banks' choice of optimal portfolio distribution between two forms of bank financing made available to consumers and entrepreneurs. In this paper, we extend that research and explore two situations under which a PLS contract may be derived. In one event, the only choice a profit-maximizing firm (entrepreneur) has for a funding is to seek out PLS financing from a profit-maximizing bank. In the other event, such a firm has the option of choosing between MU and PLS.

We use risk adjusted expected cumulative profit functions in our analysis. For either event, we derive a necessary condition for the existence of a PLS contract. Together with that, by using the concept of bargaining zones (BZ) (found in the labor relations literature on collective bargaining) we are able to identify a process under which a successful negotiation may be undertaken. Also, through comparative statics analysis, we make a series of predictions as to how the bargaining zone is likely to adjust to changes in various underlying factors. Clearly, in both our models, given the technical limits of the preferences of the two parties, and regardless of their desire to maximize their share of profits, the party with more dominant market power is likely to dictate the final deal.

The literature survey has already been included above in Section 1. In Section 2, we present our two models for the alternative situations under which a PLS contract may arise. This includes discussing the rates of return to capital and entrepreneurship. Before concluding in Section 4, we analyze technical results of the two models as well as the comparative static results in Section 3.

2. Models

In what follows, and as already mentioned, we develop two distinct models under which PLS contracts may evolve. We examine the microeconomics underlying the behavior of firms. This allows us to shed light on the nature of the contract and the process by which it is attained by creating a BZ for the profit sharing rate. In each case, a comparative statics analysis allows us to predict changes to the BZ.

2.1 Model 1: ZIFS Deposit vs. PLS-based Investment

In this model, we examine a situation when a PLS contract will be more profitable for both the entrepreneur and the bank. Henceforth, for simplicity, they are both assumed to be risk neutral. The opportunity cost faced by the bank is the sum forgone on alternative MU investment, while that faced by the entrepreneur constitutes two potential flows: earnings foregone from investing money in deposit accounts and that from the employment of knowledge, skill and abilities (aka KSA) in a similarly valued career[(3)].

The equation (1.1.a) below represents an entrepreneur's risk-adjusted expected cumulative net earnings (E^F) in T periods from bank deposit and KSA utilization (E):

$$E^F = P_B\sum_i^T fmsF/\sigma_B + (P_a/\sigma_a)\sum_i^T E_i = (P_B/\sigma_B)TfmsF + (P_a/\sigma_a)E \qquad (1.1.a)$$

Where:

(3) Imagine the project requires \$800,000 (F), of which the entrepreneur already possesses \$200,000 (i.e., s = 0.25). Suppose, also, that the bank earns a profit rate of 20% (m) on this sum of which it keeps 12% and the depositor receives 8% (i.e., f = 0.4). So, while the opportunity cost faced by the entrepreneur for investing \$200,000 is \$16,000 per year of the life-cycle of the investment, the opportunity cost to the bank for financing \$600,000 is \$120,000 per year also should the financing be reimbursed through one lump sum payment at the end of the life-cycle of the investment.

F = Total financial outlay of the firm's investment

s = Fraction of F contributed by the firm ($0 \leq s \leq 1$)

m = Rate of profit per period of the bank

f = Fraction of 'm' paid out to the depositor(s), ($0 < f < 1$)

E_i = Entrepreneur's imputed opportunity cost (known or anticipated) from activities in the i^{th} year, where $E = \sum_i^T E_i$

P_B = Probability of successful investments by the bank, $0 \leq P_B \leq 1$

σ_B = Standard deviation of number of successful investments = $\sqrt{[(1 - P_B)/n]}$ [4]

n = Number of financings and investments contracted by the bank

P_a = Probability of the entrepreneur landing employment reflecting own KSA, $0 \leq P_a \leq 1$

σ_a = Standard deviation corresponding to $P_a = \sqrt{[(p_a(1 - p_a)/n_a]}$

n_a = Number of similarly qualified individuals looking for work

T = Duration of financing contract ($1 \leq T \leq N$)

N = Duration of investment's life cycle

We start with an entrepreneur who has a fraction 's' of F, the total amount of money needed to start a business, deposited in a bank. As a depositor, he receives 'fm' fraction per dollar of his deposit as earnings where 'm' is the bank's rate of profit per period of which the bank distributes 'f' fraction to its depositors and keeps (1–f) fraction as its profit per period. Hence, in T periods, the entrepreneur's cumulative earnings from his bank deposit is $\sum_i^T fmsF = TfmsF$. However, bank earns profit only from its successful investments. If P_B is the probability of bank's successful

(4) The stochastic process adopted here is for proportion. An increase in the number of observations (e.g., financing) decreases the standard deviation. So, even with the same probability, a bank with a larger portfolio will have a lower risk.

investments, $0 \leq P_B \leq 1$, the entrepreneur's expected cumulative earnings reduces to $P_B TfmsF$. Again, different investments have varying degrees of risk and, as a result, are not comparable unless we convert all earnings as return per unit of risk. Following Sharpe (1994), we measure risk by the standard deviation of the number of successful investments, and calculate risk-adjusted expected cumulative net earnings from bank deposit as $P_B TfmsF/\sigma_B$, where σ_B is standard deviation of the bank's number of successful investments(5). Here, as with the other cases later on, a binomial process is used to derive the standard deviation. This is validated on two grounds: (i) the expectation operator is a measurement of proportion and its variability is captured by this form of standard deviation operator, and (ii) since it is a pure number it allows the objective function to retain its monetary unit ($, in this case).

While the entrepreneur's money earns return from bank deposits, he also earns E_i in the i^{th} year with his KSA so that his cumulative earning is $E = \sum_i^T E_i$. When P_a is the probability of the entrepreneur landing employment reflecting own KSA, $0 \leq P_a \leq 1$, and σ_a is the standard deviation corresponding to P_a, the risk-adjusted expected cumulative net earnings of the entrepreneur's KSA is represented as $(P_a/\sigma_a)\sum_i^T E_i = (P_a/\sigma_a)E$. Since the entrepreneur has to sacrifice his earnings from bank deposits and those rooted in utilization of KSA, equation (1.1.a) also represents an entrepreneur's opportunity cost of starting a business with his 'sF' amount of money.

Similarly, the bank's risk-adjusted expected cumulative net earnings in T periods, E^B, after paying depositors can be represented by the equation (1.1.b) below:

$$E^B = (P_B/\sigma_B)(1 - f)m(1 - s)F\{(T + v)/2\}, \; v = 1 \text{ or } T \qquad (1.1.b)$$

where $(1-s)F/T$=Per period reimbursement of borrowed capital as well as the final payment amount under periodic payment scheme, and v=1, or T: accordingly as financing payment follows an arithmetic series over T periods, or is one-time lump sum payment at final period T.

(5) An example is best for this purpose. For $500m expected profit, if risk is 0.5, then for each unit of risk, the expected profit is $1,000m. If the risk is 0.25 and expected profit, $250m, then for each unit of risk, the expected profit is $1,000m. So, a risk-neutral entity will be indifferent between the two choices.

Here, the bank finances (1–s)F to the entrepreneur. In turn, based on profit earned, it retains (1–f)m per dollar of so that the bank's net earnings against (1–s)F amount of financing is (1–f)m(1–s)F per period. As P_B is the probability of successful investments and σ_B is its standard deviation, the bank's risk-adjusted expected cumulative projected net earnings for the duration of the financing is given by $(P_B/\sigma_B)\sum(1\text{-}f)m(1–s)F$. We refer to it as RAECP.

The firm may reimburse its debt (principal and MU payment) to the bank as a constant sum installment over the contract duration, or pay-off all with a one-time payment at the end of the contract period. Thus, the RAECP in T periods is $(P_B/\sigma_B)(1\text{-}f)m(1–s)FT$ if a one-time payment is made, and it is $(P_B/\sigma_B)(1\text{-}f)m(1–s)F(T+1)/2$ if it is paid in installments. This is represented in equation (1.1.b). Since we are calculating the net earnings of the bank, equation (1.1.b) deals with the firm's MU payment only(6). Note that E^F and E^B also represent the opportunity costs of the firm and the bank, respectively, of a PLS investment.

Following Siddiqi (1988), if the firm and the bank agree to make a PLS contract for T periods, the respective retained portions of RAECP of the firm ($\Pi_{PLS}{}^F$) and the bank ($\Pi_{PLS}{}^B$) can be written as:

$$\Pi_{PLS}{}^F = (1 - \lambda)P_{PLS}\sum_i{}^T(P_i – AFC_i – AVC_i) \text{ x } Q_i/\sigma_{PLS} = (1 – \lambda)(P_{PLS}/\sigma_{PLS})K \quad (1.2.a)$$

$$\Pi_{PLS}{}^B = P_{PLS}(\delta\sum_i{}^T(P_i – AFC_i – AVC_i) \text{ x } Q_i –\sum_i{}^Tt_i\})/\sigma_{PLS} = (P_{PLS}/\sigma_{PLS})(\delta K – t) \quad (1.2.b)$$

where (P_i, Q_i, AFC_i, AVC_i) = Output price; Output volume; Average Fixed Cost; Average Variable Cost in the i^{th} year. Here, AFC_i includes the opportunity cost of the bank and the firm's financial capital as well as the opportunity cost of entrepreneurship in the i^{th} year,

∑iT(P – AFCi – AVCi) x Qi = K = Cumulative economic profit over the duration of PLS contract; K > 0 (for both parties to be inclined toward a PLS contract),

(6) With constant periodic payment, we have a diminishing arithmetic series of the debt (1–s)F being repaid to the bank by the firm, with an arithmetic difference of (1–s)F/T, with the initial value of (1–s)F and the final value of (1–s)F/T. Thus, we get, (1–s)F{(T/2)(1+1/T)} = (1–s)F(T+1)/2. Under one-time payment arrangement, we get (1–s)F(T+T)/2= (1–s)FT. So, in the first case, v=1 and in the second case, v=T.

λ = Fraction of the firm's profit that the firm is willing to offer to the bank for bank financing,

δ = Fraction of the firm's profit that the bank is willing to accept for bank financing,

t_i = Monitoring cost incurred by the bank in the i^{th} year ($\sum t_i^T = t$, total cost for lifetime of PLS contract),

P_{PLS} = Probability of successful PLS investment, $0 \leq P_{PLS} \leq 1$,

σ_{PLS} = Standard deviation corresponding to $P_{PLS} = \sqrt{\frac{P_{PLS}(1-P_{PLS})}{n_{PLS}}}$, and

n_{PLS} = Number of PLS financings contracted by the bank

One can expect that, for the bank, the probability of success is lower for PLS investment compared with MU investment, $P_{MU}^B > P_{PLS}$. The corresponding standard deviations may be expected to relate in the opposite order, $\sigma_{MU}^B < \sigma_{PLS}$ for the right hand side segment of the distribution(7). In such circumstance, in order to be not cheated or misled by the firm causing things to be worse than they really are, the bank might be interested in monitoring firm's activities so as to minimize MH. Of course, such monitoring does come with a cost as indicated by the adoption of the variable, t_i.

Now, a risk-neutral firm will be indifferent between deposits in a ZIFS account and PLS financing, or prefer the latter, when $E^F \leq \Pi_{PLS}^F$. Here, we assume a *muḍārabah* arrangement where the bank is not authorized to interfere in the routine transaction of the business, but is empowered to audit the accounts and acquire information regarding important decisions taken by the entrepreneur (Siddiqi, 1988). Thus, based on equations (1.1.a) and (1.2.a), we get:

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(7) In this stochastic process, σ is symmetrically distributed. However, as P increases from 0.5 to 1.0, σ decreases. One could use 0.5 as a cut-off point arguing that no investment should rationally be undertaken that has not a better than 50-50 chance of success. But that is not entirely true. Given that we adjust expected cumulative earnings and profit for underlying risk, the outcome is amenable to be chosen by firms and banks with risk profile running from utter aversion to abject adoration.

$(P_B/\sigma_B)TfmsF + (P_a/\sigma_a)E \leq (1 - \lambda)(P_{PLS}/\sigma_{PLS})K$

Or $\lambda \leq [(P_{PLS}/\sigma_{PLS})K - (P_B/\sigma_B)TfmsF - (P_a/\sigma_a)E)]/[(P_{PLS}/\sigma_{PLS})K]$ (1.3.0)

With equality in (1.3.0), we get:

$\lambda' = [(P_{PLS}/\sigma_{PLS})K - (P_B/\sigma_B)(TfmsF) - (P_a/\sigma_a)E)]/[(P_{PLS}/\sigma_{PLS})K]$, which is the firm's maximum possible bid-rate to the bank. It implies that $0 \leq \lambda \leq \lambda'$.

Again, a risk-neutral bank will be indifferent between being an MU creditor and PLS creditor, or prefer the latter, when $E^B \leq \Pi_{PLS}{}^B$. So, based on equations (1.1.b) and (1.2.b), we get:

$(P_B/\sigma_B)(1 - f)m(1 - s)F\{(T + v)/2\} \leq (P_{PLS}/\sigma_{PLS})(\delta K - t)$

Or $\delta \geq [(P_B/\sigma_B)(1 - f)m(1 - s)F\{(T + v)/2\} + (P_{PLS}/\sigma_{PLS})t]/[(P_{PLS}/\sigma_{PLS})K]$ (1.4.0)

With equality in (1.4.0), we get:

$\delta' = [(P_B/\sigma_B)(1 - f)m(1 - s)F\{(T + v)/2\} + (P_{PLS}/\sigma_{PLS})t]/[(P_{PLS}/\sigma_{PLS})K]$

Thus, with $\delta' \leq \delta \leq 1$, δ' is the bank's minimum acceptable ask-rate of the firm.

For a contract to arise, two parties must agree to a common or equilibrium share, i.e., $\lambda_E = \delta_E$. This means, using equations (1.3.0) and (1.4.0), we get:

$[(P_{PLS}/\sigma_{PLS})K - (P_B/\sigma_B)(TfmsF) - (P_a/\sigma_a)E)]/[(P_{PLS}/\sigma_{PLS})K] \geq \lambda_E = \delta_E$

$\geq [(P_B/\sigma_B)(1 - f)m(1 - s)F\{(T + v)/2\} + (P_{PLS}/\sigma_{PLS})t]/[(P_{PLS}/\sigma_{PLS})K]$ (1.5.0)

Based on inequality (1.5.0), we are able to derive a necessary condition for a contract to exist at all. Since λ' is the firm's maximum bid-rate and δ' is the bank's minimum ask-rate, it is clear that no PLS contract is possible when $\delta' > \lambda'$ because it causes a divergence of the individually acceptable settlement zones. So, a necessary condition for a PLS contract to be successfully negotiated is: $\lambda' \geq \delta'$. This means:

$[(P_{PLS}/\sigma_{PLS})K - (P_B/\sigma_B)(TfmsF) - (Pa/\sigma a)E)] \geq$

$[(P_B/\sigma_B)(1 - f)m(1 - s)F\{(T + v)/2\}\} + (P_{PLS}/\sigma_{PLS})t]$

Or $(P_{PLS}/\sigma_{PLS})(K - t) \geq E^F + E^B$ (1.5.1)

That is, the risk and monitoring cost adjusted expected cumulative profit of the firm must at least be equal to the sum of the firm and the bank's opportunity costs of PLS investment. This result is simple to understand. The net income from PLS contracts must be higher or at least equal to the income from before PLS contract net income. This necessary condition has two special cases:

1. With implicit trust, i.e., complete cooperation, transparency and honesty, leading to t = 0, the necessary condition $\lambda' \geq \delta'$ translates to (1.5.1a) below:

$$(P_{PLS}/\sigma_{PLS})K \geq E^F + E^B \qquad (1.5.1a)$$

Here, $\delta' = [(P_B/\sigma_B)(1 - f)m(1 - s)F\{(T + v)/2\}]/[(P_{PLS}/\sigma_{PLS})K]$.

2. If the potential entrepreneur launching the firm contributes no capital except for the idea, the drive, and the organizational capacity behind the enterprise, then s = 0. With monitoring cost still existing, the necessary condition reduces to:

$$(P_{PLS}/\sigma_{PLS})(K - t) \geq (P_a/\sigma_a)E + [(P_B/\sigma_B)(1 - f)mF\{(T + v)/2\}] \qquad (1.5.1b)$$

i.e., the appropriately weighted difference of RAECP and the bank's monitoring cost has to be at least equal to the sum of opportunity costs to the entrepreneur on KSA grounds and the bank for financing the entirety of PLS project. However, this is more in keeping with the situation wherein an innovator is financed by a venture capitalist and is rewarded for it with a share of the business and a corresponding share of the profit.

Here,

$$\lambda' = [(P_{PLS}/\sigma_{PLS})K - (P_a/\sigma_a)E)]/[(P_{PLS}/\sigma_{PLS})K]$$

and

$$\delta' = [(P_B/\sigma_B)(1 - f)mF\{(T + v)/2\} + (P_{PLS}/\sigma_{PLS})t]/[(P_{PLS}/\sigma_{PLS})K]$$

Bargaining Zone

If the necessary condition is satisfied, profit maximizing behavior will lead the firm to try to settle as close to δ' as possible, whereas the bank will try to reach as high as λ'. Let λ_b be the initial bid-rate posed by the firm to the bank and let δ_a be the initial ask-rate posed to the firm by the bank in the bargaining phase. Further, as to the process of negotiation, as long as $\lambda_b < \lambda'$ and $\delta_a > \delta'$ [(8)] then it should be possible to negotiate a contract without possible impasse[(9)].

Even though MU and PLS related stochastic parameters as well as K are known to both parties, E, t and P_a and σ_a are not shared information. Thus, with asymmetric information, the bargaining limits of both parties are not known to one another. This situation is depicted in Figure 1 as the BZ. Note that while it is depicted that $\delta_a \leq \lambda'$ and $\lambda_b \geq \delta'$, there is no reason why it cannot be that $\delta_a > \lambda'$ and $\lambda_b < \delta'$. However, the final negotiated (equilibrium) rate given by $\lambda_E = \delta_E$ must lie in the closed range (λ', δ').

Unlike the above, under the exceptional case of symmetrically shared information, it would appear that at the outset of negotiation $\lambda_b = \delta'$, and $\delta_a = \lambda'$. Thus, one would expect that mutual discovery process of positions and the bargaining duration involved would be abbreviated.

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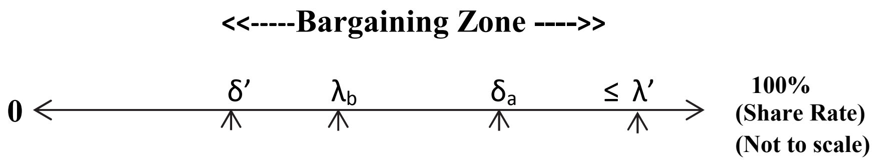
(8) If $\lambda' < \delta'$, yet even with $\lambda b \leq \lambda'$ and $\delta a \geq \delta'$, the two parties' choices will not converge. They will diverge leading to no contract being drawn up.

(9) As to the process of bargaining, even with the necessary condition holding, the process may break down if either of the negotiating party is naïve enough to commence bargaining by positing their extreme position. In that case, should the other party prove intransigent with the offer; the offending party will squeeze itself out of room to compromise and strike a deal.

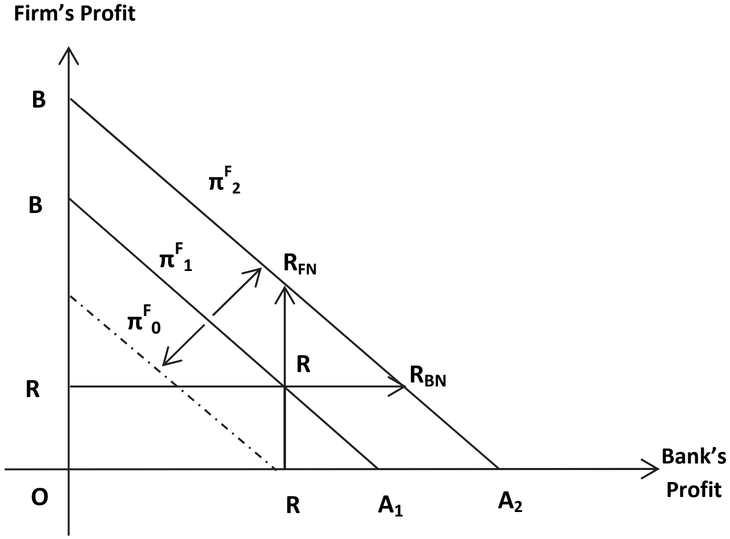
Figure (1). Firm & Bank Negotiate PLS Contract for Bank's Share of Profit under Asymmetric Information.



A more comprehensive view of λ' and δ' and the BZ is depicted in Figure 2. It also allows us to view how these three elements change when either of the opportunity costs (reservation profits) of the firm or the bank, or the firm's RAECP changes. The latter part ties in with the Comparative Statics Analysis in Section 3.

In Figure (2), OR_F is the minimum amount of firm's RAECP that firm requires to enter into a PLS contract, (the firm's reservation profit). Similarly, bank's reservation profit is given by OR_B. Now, A_1B_1 is one of the iso-profit lines in an iso-profit map representing firm's RAECP, πF_1. Here, higher iso-profit lines represent higher levels of profit e.g., A_2B_2 represents πF_2, where $\pi F_1 < \pi F_2$. When firm's profits are $\pi^F{}_1$ and $\pi^F{}_2$, we calculate $\lambda'_1 = R_FB_1/OB_1$ and $\lambda'_2 = R_FB_2/OB_2$, respectively. Observe that as profit increases, λ' increases as we get $\lambda'_1 < \lambda'_2$. For the bank, $\delta'_1 = OR_B/OA_1$; $\delta'_2 = OR_B/OA_2$ and as firm's profit increase, the ratio of reservation profit of the bank to firm's profit decreases, i.e., $\delta'_1 > \delta'_2$. Thus, with individual opportunity cost fixed, as profit increases the firm wants to give relatively more to the bank and the bank wants relatively less for itself from the firm. So, both parties behave not only rationally, but also identically.

Figure (2). Determination of the Necessary Condition for the Existence of a PLS Contract given the Reservation Profits of the Firm and the Bank.



We have already determined that for a PLS contract to exist, λ' has to be at least as big as δ', i.e., $OR_F \leq OR_B$. Here, we derive a second necessary condition regarding firm's RAECP. It has to be at least $\pi^F{}_1 = OR_F + OR_B$, the profit with the iso-profit line A_1B_1 on which the reservation point, R, with coordinates (R_B, R_F) lies. At that point, $\lambda' = \delta'$. For any profit greater than πF_1, e.g., πF_2, the bargaining zone is the closed range between R_{FN} and R_{BN}.

As to the comparative statics results, any change in a parameter either changes the opportunity costs (changes in m, F, f, T, s, P_B, σ_B, E, P_a, or σ_a) or changes profit (changes in K, t, P_{PLS}, or σ_{PLS}) (Equation (1.1.a), (1.1.b), (1.2.a), (1.2.b)). We already mentioned that an increase in profit increases λ' and reduces δ'. So far as the opportunity costs are concerned, any increase in E^F due to a change in a parameter will reduce λ', while any increase in E^B will increase δ' leading to all our comparative statics results in Section 3. This is because an increase in the opportunity cost will reduce the attractiveness of a PLS contract inducing firms to reduce the upper limit of the bid-rate and the bank to increase the lower limit of the ask-rate.

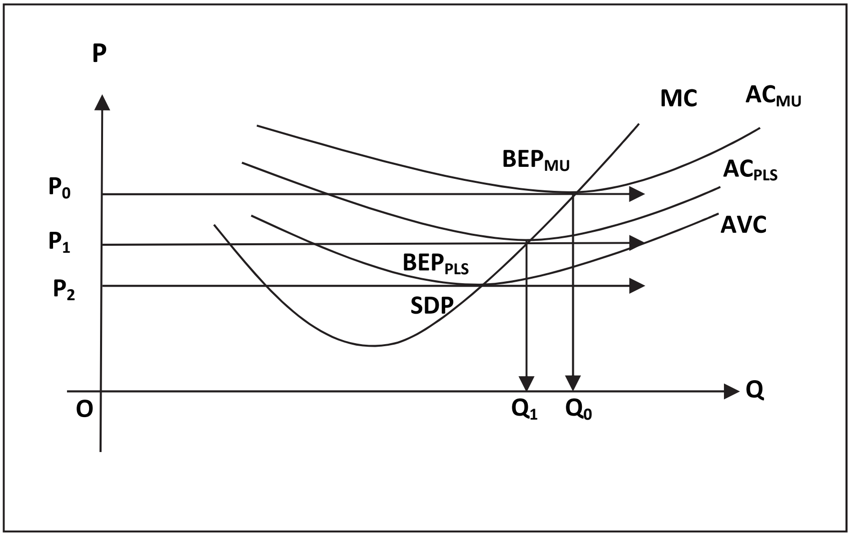
2.2 Model 2: MU Entrepreneurship vs. PLS Entrepreneurship

In this model we examine the question as to when a firm chooses the PLS mode over the alternative MU mode when the latter is also available. In order to derive a necessary condition similar to $\lambda' \geq \delta'$ in Model 1, here, the microeconomic basis of this variation has to be established first.

Consider a firm with the option of seeking financing under MU and PLS labeled as F_{MU} and F_{PLS}, respectively. The cost structure for F_{MU} is represented in Figure 3 based on profit maximization models under MU and PLS, by AVC, ATC_{MU} and marginal cost, MC. The shutdown and breakeven points are respectively shown by SDP and BEP_{MU} with the efficient scale of output (Q_0). With F_{PLS}, the only thing that would change structurally would be the average fixed cost leading to a change in the average total cost curve. It would drop down to a new position, ATC_{PLS}. Consequently, two things change: the breakeven point (BEP_{PLS}) and the efficient scale of output (Q_1). This is explained by the fact that since mark-up payment (MUP) is independent of the amount of production, it is a part of fixed cost accruing for F_{MU}. For F_{PLS}, this component is absent and as a result $AFC_{MU} > AFC_{PLS}$ by the amount of total mark-up payment. On the other hand, average variable cost remains the same under two different financing arrangements i.e., $AVC_{MU} \equiv AVC_{PLS} = AVC$ (say) which implies $ATC_{MU} > ATC_{PLS}$. It means that while the shut-downs points are identical ($SDP_{MU} \equiv SDP_{PLS} = SDP$), the break-even points (BEP_{MU} and BEP_{PLS}) differ under the two different financial arrangements. Since for all levels of output, $ATC_{MU} > ATC_{PLS}$, $BEP_{MU} > BEP_{PLS}$, hence, F_{PLS} is less sensitive to a downward price change than F_{MU}.

It also means that the efficient scale output for F_{MU} will exceed that for F_{PLS}, i.e., $Q_0 > Q_1$. In other words, smaller markets or smaller market shares should pose less of a problem to F_{PLS} when compared with F_{MU}. Thus, in emerging economies as well as for emerging firms, the financing alternative offered by PLS holds promise.

Figure (3). Profit Maximization Model under MU and PLS Credit.



Since AVC and the corresponding MC are identical for firms financed in either of the two ways, the profit-maximizing production decisions are same under the two modes of financing. Thus, faced with the same market equilibrium price, P_0, $Q_{MU} \equiv Q_{PLS} = Q_0$. With the same scale of operation, F_{MU} will break even while F_{PLS} will make positive economic profit. Hence, its viability is better assured owing to the structural shift based on financing change and not due to technical change. Furthermore, in the long-run, with any price below P_0 but greater than or equal to P_1, F_{PLS} break-even price, F_{PLS} will survive but F_{MU} will not. Thus, the price at which PLS financing firm breaks even, MU financing firms earn economic losses and go out of business in the long-run. Hence, under PLS, the probability of survival is higher.

However, this illustration does not capture the entire story. When price is less than P_1 and greater than or equal to P_2, F_{PLS} is not breaking even, but unlike the case with F_{MU}, it is not operating alone and does not have to bear the risk due to the loss all by itself. Its obligations are mitigated by PLS contracts. In this event, a part of per unit difference between P_1 and P_2 is absorbed by the bank. Thus, this element of shared risk further enhances the firm's viability.

Using the same cumulative function (∑) used in Model 1 to capture the typical multiple-year spread of financing, with the equivalent Q already established above, the respective RAECP function under MU and PLS financing for the firm and the bank may be written as:

$\Pi_{MU}^{F} = P_{MU}^{F}\sum_i^T(P_i – AFC_{MU,i} – AVC_i) \times Q_i/\sigma_{MU}^{F} = (P_{MU}^{F}/\sigma_{MU}^{F})\ (K– \{(T+ v)/2\}MUP)$ (2.1.a) [10 11]

$\Pi_{MU}^{B} = P_{MU}^{B}\sum_i^T(AFC_{MU,i} – AFC_{PLS,i})/\sigma_{MU}^{B} = (P_{MU}^{B}/\sigma_{MU}^{B})\{(T + v)/2\}MUP$ (2.1.b)

$\Pi_{PLS}^{F} = (1 – \lambda)P_{PLS}\sum_i^T(P_i – AFC_{PLS,i} – AVCi) \times Q_i/\sigma_{PLS} = (P_{PLS}/\sigma_{PLS})(1 - \lambda)K$ (2.2.a)

$\Pi_{PLS}^{B} = [P_{PLS}\{\delta\sum_i^T(P_i – AFC_{PLS,i} – AVC_i) \times Q_i – \sum t_i\}]/\sigma_{PLS} = (P_{PLS}/\sigma_{PLS})(\delta K – t)$ (2.2.b)

Here, MUP is the mark-up payment, $(AFC_{MU,i} – AFC_{PLS,i})$, for all i, made by the firm to the bank in an MU contract. All other variables are the same as those defined under equations (1.2.a) and (1.2.b) in Model 1. With the probability of firm's success higher under PLS financing than under MU financing, $P_{MU}^{F} < P_{PLS.}$ Also, since the firm doesn't have to pay to the bank in case of a loss under PLS financing, we can argue that the risk is larger for F_{MU} than for F_{PLS}, i.e., $\sigma_{MU}^{F} > \sigma_{PLS}$. Again, from the bank's perspective, the probability of earning return from investment is higher under MU arrangement, i.e., $P_{MU}^{B} > P_{PLS.}$ Since not only the firm does not have to pay to the bank in case of a loss under PLS financing, whatever loss is incurred is shared with the bank, we can argue that the risk is larger for the bank under PLS arrangement, i.e., $\sigma_{MU}^{B} < \sigma_{PLS}$. Hence, we get, $P_{MU}^{F} < P_{PLS} < P_{MU}^{B}$, while $\sigma_{MU}^{F} > \sigma_{PLS} > \sigma_{MU}^{B}$.

A firm will be indifferent between MU financing and PLS financing, or prefer the latter, when $\Pi_{MU}^{F} \leq \Pi_{PLS}^{F}$. Thus, combining inequalities (2.1.a) and (2.2.a), we get,

$(P_{MU}^{F}/\sigma_{MU}^{F})(K – \{(T + v)/2\}MUP] \leq (P_{PLS}/\sigma_{PLS})(1 - \lambda)K$

Or $\lambda \leq [(P_{PLS}/\sigma_{PLS})K - (P_{MU}^{F}/\sigma_{MU}^{F})[K – \{(T + v)/2\}MUP]]/[(P_{PLS}/\sigma_{PLS})K$ (2.3.0)

(10) Here, in equation (2.1.a), ∑iT(Pi – AFCMU,i – AVCi) x Qi has been rewritten as: ∑iT(Pi-AFCPLS,i–AVCi) – (AFCMU,i–AFCPLS,i)) x Qi = K-∑iT(AFCMU,i–AFCPLS,i). Then, ∑iT(AFCMU,i – AFCPLS,i) is written as {(T + v)/2}MUP.

(11) Diminishing arithmetic series over T periods (T ≤ N); initial value (AFCMU – AFCPLS) and with (1/T)th as its final value.

Here, $\lambda' = [(P_{PLS}/\sigma_{PLS})K - (P_{MU}{}^{F}/\sigma_{MU}{}^{F})[K - \{(T + v)/2\}MUP]]/[(P_{PLS}/\sigma_{PLS})K$ is the firm's maximum bid-rate; i.e., $0 \leq \lambda \leq \lambda'$.

The bank, on the other hand, will be indifferent between financing F_{MU} and F_{PLS}, or prefer the latter, when $\Pi_{MU}{}^{B} \leq \Pi_{PLS}$. Thus, combining inequalities (2.1.b) and (2.2.b), we get,

$$(P_{MU}{}^{B}/\sigma_{MU}{}^{B})\{(T + v)/2\}MUP \leq (P_{PLS}/\sigma_{PLS})(\delta K - t)$$

Or $$\delta \geq [(P_{MU}{}^{B}/\sigma_{MU}{}^{B})\{(T + v)/2\}MUP + (P_{PLS}/\sigma_{PLS})t]/[(P_{PLS}/\sigma_{PLS})K] \qquad (2.4.0)$$

Here, $\delta' = [(P_{MU}{}^{B}/\sigma_{MU}{}^{B})\{(T + v)/2\}MUP + (P_{PLS}/\sigma_{PLS})t]/[(P_{PLS}/\sigma_{PLS})K]$ is the bank's minimum ask rate; i.e., $\delta' \leq \delta \leq 1$.

We know that a PLS contract is possible only when both parties agree to a particular share, i.e., $\lambda_E = \delta_E$. A necessary condition for this to happen is: $\lambda' \geq \delta'$ is where its inequality ensures the existence of a BZ (as in Figure 1). Based on inequalities (2.3.0) and (2.4.0), we specify below the necessary condition for a PLS contract:

$$[(P_{PLS}/\sigma_{PLS})K - (P_{MU}{}^{F}/\sigma_{MU}{}^{F})[K - \{(T + v)/2\}MUP]/[(P_{PLS}/\sigma_{PLS})K$$

$$\geq [(P_{MU}{}^{B}/\sigma_{MU}{}^{B})\{(T + v)/2\}MUP + (P_{PLS}/\sigma_{PLS})t]/[(P_{PLS}/\sigma_{PLS})K]$$

Or $$[(P_{PLS}/\sigma_{PLS})K - (P_{MU}{}^{F}/\sigma_{MU}{}^{F})[K - \{(T + v)/2\}MUP]$$
$$\geq [(P_{MU}{}^{B}/\sigma_{MU}{}^{B})\{(T + v)/2\}MUP + (P_{PLS}/\sigma_{PLS})t]$$

Or $$(P_{PLS}/\sigma_{PLS})(K - t) \geq \Pi_{MU}{}^{F} + \Pi_{MU}{}^{B} \qquad (2.5.0)$$

i.e., the RAECP of the firm under PLS after deducting monitoring cost must be at least equal to the sum of the firm's RAECP and the bank's risk-adjusted mark-up profit under MU.

Again, with complete trust, cooperation, and transparency, i.e., $t = 0$, this necessary condition boils down to:

$$(P_{PLS}/\sigma_{PLS})K \geq \Pi_{MU}{}^{F} + \Pi_{MU}{}^{B} \qquad (2.6.0)$$

Thus, for a PLS contact to be possible, the firm's RAECP under PLS must be at least equal to the sum of the firm's RAECP and the bank's risk-adjusted mark-up profit under MU. As to the BZ, whether under symmetric or asymmetric information, the earlier figures (1) and (2) still apply.

Profit Rate, Rate of Return to Invested Inputs, 50-50 Split

By establishing the BZ, we designate the potential range for the profit-sharing rate between the bank and the firm. In this section, we establish the relationship between the profit-sharing rate and the rates of return to the two basic inputs: financial capital, and KSA. Under zero economic profit (K = 0), the rates of return are same under PLS as they are under MU. However, with K > 0, we would like to know by how much those rates of return to financial capital and entrepreneurship exceed their opportunity costs. So, elucidating these rates makes our modeling more meaningful and relevant.

We have already established that in equilibrium λ_E and $(1 - \lambda_E)$ are the respective share rates of profit to the bank and the firm in excess of corresponding opportunity costs. If $\lambda_E = \delta'$, then the bank is just breaking even, i.e., earning zero economic profit, while the firm enjoys positive economic profit. This situation is completely reversed if $\lambda_E = \lambda'$. Again, the bank's share rate, λ, is solely the profit rate to its financial capital, while $(1 - \lambda)$ is the sum of the profit rates to the firm's financial capital and entrepreneurship.

If the share rate of profit accruing to financial capital is represented by c, then $(1 - c)$ is the share rate profit to entrepreneurship. While c is shared between the firm and the bank in proportion to their financial capital contributions s and $(1 - s)$, $(1 - c)$ belongs entirely to the firm. Below, we formalize the relationship between institutions (firm and bank) and between inputs (capital and entrepreneurship) in how profit is shared: bank against capital, and firm against capital and entrepreneurship in identities (3.1.a) and (3.1.b), respectively.

$$\lambda_E \equiv (1 - s)c \tag{3.1.a}$$

$$(1 - \lambda_E) \equiv sc + (1 - c) \tag{3.1.b}$$

From identity (3.1.a), we can derive the share of the profit to capital,

$$c = \lambda_E/(1 - s).$$

Now, profitability is best measured using the rate of return to investment, and not just by the absolute amount of accrued profit. The rate of return to financial capital is the same for both the firm and the bank.

$$ROR^C = cK/F = \lambda_E K/(1 - s)F \qquad (3.2.0)$$

At this rate of return, for their capital contributions, the bank and the firm respectively receive absolute sums of $\lambda_E K$ and $[\lambda_E K/(1 - s)F] \times sF = \lambda_E Ks/(1 - s)$. We already know that the rate of return to the firm for entrepreneurship is $(1 - c)$. Hence, the firm's share of absolute profit for its entrepreneurial role is given by:

$$(1 - c)K = [1 - \lambda_E/(1 - s)]K.$$

Now, if λ is set externally at, say, λ0, the problem of choosing λ for profit maximization translates into choice of s. (A regulatory body may be empowered to impose such a requirement if it so chooses.) Identities (3.1.a) and (3.1.b) imply that in case of a fixed λ, i.e., λ0, the bank will be inclined to reduce its risk by maximizing c, the return to capital, which can be achieved by increasing s. For the firm, however, maximizing (1–c), the return to entrepreneurship, (i.e., minimizing c) by decreasing s is the new choice. The basis of these choices is given by Equation (3.3.b).

$$c = \lambda_0/(1 - s) \qquad (3.3.a)$$

where:

$$\partial c/\partial s = \lambda_0/(1 - s)^2 > 0 \qquad (3.3.b)$$

As an example, we assume two choices of fixed λ: $\lambda_0 = 0.5$, and $\lambda_0 = 0.4$ as explained in Table 1. This works so long as $\delta' \le \lambda_0 \le \lambda'$. In this case, no matter how the financial contribution of the firm, s, is varied, the profit share between the firm and the bank remains at 50-50 and 60-40, respectively. Increase in s only increases c, while decreasing the risk to the bank.

Now, suppose the firm expects at least (1–c) = 0.20 for its entrepreneurial contribution based on its opportunity cost. Then with $\lambda_0 =$ 0.5, it will be willing to provide at most s = 0.375 as its share of capital. When λ0 = 0.4, on the other hand, given the same expected minimum (1– c), the firm's maximum possible financial contribution rises to 50%.

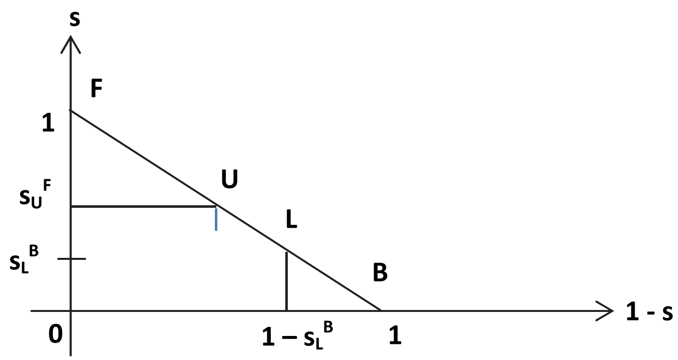
This example suggests that when λ is given, there exists an upper limit to firm's willingness to contribute capital (s_u^F). The bank also imposes a minimum capital contribution requirement on the firm (s_l^B) so as to reduce adverse selection during contract signage and moral hazard later. In other words, it acts as a self-selection or as a rationing device.

Table (1). Fixed λ, and Relationship between 's' and 'c'.

λ = 0.5			λ = 0.4		
s	c = λ/(1 - s)	1 - c	S	c = λ/(1 - s)	1 - c
0	0.50	0.50	0	0.40	0.60
0.1	0.56	0.44	0.1	0.44	0.56
0.375	0.80	0.20	0.50	0.80	0.20

Figure (4) illustrates the range for s that arises when λ is given. As mentioned above, it cannot exceed s_U^F or fall short of s_L^B. The former is the most the firm is willing to contribute, whereas the latter is the least the bank will accept from the firm as a contribution. Thus, with an imposed λ_0, the final values of s and (1–s) will fall in UL segment of FB.

Figure (4). Fixed λ, and Relationship between 's' and 'c'.



Now, at the outset we considered λ as the choice variable over which to contract. Also, above, we saw that when λ is given, s becomes the choice variable. Below, we deal with the situation wherein $c = c_0$ is externally imposed. Of course, given the relationship (3.1.a), we can claim,

$$c^{max} = \lambda'/(1 - s) \quad (3.4.a)$$
$$c^{min} = \delta'/(1 - s) \quad (3.4.b)$$

So, c_0 must fall in the range given by equations (3.4.a) and (3.4.b) in order for a financing to be contracted. Now, following Siddiqi (1988), should capital's share of the profit equal 0.5, then $c^{min} \leq 0.5 \leq c^{max}$. In Table 2.0 below, we use identities (3.1.a) and (3.1.b) to illustrate the relationship between s and λ_E when c is held constant. In complete contrast to the earlier case of holding λ constant, profit share accruing to the bank is inversely impacted by changing s. On the other hand, for the firm, it is directly impacted. Hence, given the advantages of other limits, unlike in Table 1, the bank proposes to reduce firm's down payment, while the firm seeks to increase it.

Table (2). Fixed c, and Relationship between 's' and 'λ_E'.

	$c_0 = 0.5$		$c_0 = 0.4$	
	Profit Share			
s	Firm	Bank	Firm	Bank
0	0.50	0.50	0.60	0.40
0.20	0.60	0.40	0.68	0.32
0.80	0.90	0.10	0.92	0.08

3. Analysis of Results

One of the important contributions of our paper is to analyze the impact of AS and MH on the part of the firm on the nature of PLS contracts, and the subsequent distribution of the profit stream between the two parties. While ours is a model with symmetric information between the firm and the bank with respect to the distributional parameters, that may not be the case with K ($\equiv \sum_i^T(P_i - AFC_i - AVC_i)$), the projected cumulative profit stream. While applying for financing, the firm might purposefully misrepresent its actual K. That would constitute AS. As to the behavior of the firm that leads to AS, inequalities (1.3.0) and (1.4.0) give us some insight. It is not in the firm's best interest to under-represent K during the application and evaluation phase, as that will lead to a higher ask-rate (δ') from the bank. Underreporting of K might push δ' higher than λ' when the necessary condition for a contract will be violated. In that case, the firm will not be able to secure PLS financing.

The firm will have good reasons to over-report K, however. If this falsification is accepted as such by the bank, then δ' will be lower. However, should the contracted $\lambda_E = \delta_E$ settle lower than it would be in the absence of falsification, the bank would lose. In addition, if this is followed by MH whereupon the firm can further under-report the actual profit during operation, the bank will earn even less thereby become a double-loser.

One way to reduce the latter form of cheating is for the bank to monitor profit of the firm. A strict legal mechanism sufficiently punitive can elicit honesty on the part of the firm post contract. Also, if a firm wants to be a repeat borrower with its bank, then an honest track record would likely improve the trust of the bank thereby reducing the monitoring cost, t, faced by the bank (equations (1.4.0) and (2.4.0)). This will reduce the bank's minimum ask-rate, δ', which, in turn, make the firm gainer in the long run.

Through MH the firm will also be able to underpay taxes to the government. However, since PLS financing essentially leads to joint liability, this is likely to jeopardize the bank's good standing with the government unless it can prove that it was cheated also. Of course, the firm can collude with the bank and under-report its profit stream, thereby defrauding only the government. So, the firm may have to do cost-benefit analysis of cheating the bank vs. cheating the government vs. cheating them both. The bank will have to do the same before it wants to collaborate with the firm to get into an illegal action.

Question arises here whether the amount of contribution expected of the firm in its total investment (i.e., sF) could mitigate or prevent misleading behaviors producing AS, MH, or both? We are ruling out examining the rare case of complete fraud whereupon somehow the firm is able to abscond with a significant fraction of the money financed by the bank such that it exceeds its own share of total investment. Typically, this could only arise by virtue of complete procedural failure on the part of the bank. Our interest is to examine if and how the bank may be able to reduce or prevent a systematic under-reporting of profit by the firm as well as cause the firm to rectify any clear source of operational inefficiency.

When a business fails regardless of whether it is due to incompetence or obfuscation, the borrowed money that goes into operation and not into fixed capital acquisition is not salvageable unless it has not been spent. Now, in case of a business failure, the firm and the bank will have to divide up the salvaged sum in an agreed upon manner. Suppose, given the depreciation rate of fixed capital (FC, where FC $\leq$ F) and given the time frame at which the business goes bankrupt, a fraction α, ($0 \leq \alpha < 1$) of FC may be salvaged. Out of αFC recovered amount, the firm and the bank will receive αsFC and $\alpha(1 - s)$FC respectively unless the failure can be attributed to the negligence or unethical behavior on the part of the management. That means the greater is FC, the more the bank is likely to recover in case of business failure. So, the total financial need, F, should be carefully assessed between FC and operating expenses so as not to be unusually generous.

The changes to BZ that likely arise through changes in λ' and δ' due to parametric shifts are explained below. However, the actual *comparative statics* results are included in Appendix I and Appendix II.

Model 1 Comparative Statics Results(12)

We found that an increase in P_B, T, f, m, s, F, and E or P_a which increases the opportunity cost of the firm of a PLS investment (Equation 1.1.a) reduces the upper limit of the bid-rate (λ'), and an increase in σ_B or σ_a that reduces opportunity cost increases λ'. On the other hand, the lower limit of the ask-rate (δ') is positively related with the opportunity cost of the bank of a PLS contract (Equation 1.1.b); i.e., an increase in P_B, m, F, or T increases δ', whereas an increase in f, s, and σ_B reduces δ'. This is because higher opportunity cost makes both parties more conservative in their bid/ask rates. We also found that RAECP of the firm ($\Pi_{PLS}{}^{F}$) is positively related with λ' and that of the bank is negatively related with δ'. This is because higher expected profit allows both parties willing to accept a little lower share. Results above imply that an increase in m, F,

(12) See Appendix I.

T, t, P_B, or σ_{PLS} will reduce the bargaining zone (BZ), an increase in K, σ_B, or P_{PLS} will increase the BZ, whereas the effect of an increase in f, s, or a on BZ is indeterminate.

Model 2 Comparative Statics Results[13]

Results are almost identical to that in Model 1. The opportunity cost of the firm undertaking a PLS investment (Equation 2.1.a) is inversely related to λ'. So, as opportunity costs are increased by $P_{MU}{}^{F}$ and K and decreased by $\sigma_{MU}{}^{F}$, T and MUP, they are, accordingly, negatively and positively correlated with λ'. On the other hand, for the bank, δ' is positively related with the opportunity cost of a PLS contract (Equation 2.1.b); i.e., an increase in $P_{MU}{}^{B}$, T, or MUP increases δ'; while an increase in $\sigma_{MU}{}^{B}$ reduces δ'.

An increase in P_{PLS} increases firm's profit thereby increasing λ'. The only exception to our rule is that an increase in K increases firm's profit but reduces λ'. This is because an increase in K also increases the opportunity cost of the firm of a PLS investment which is the dominant force in influencing λ'. However, as in Model 1, K is negatively co-related with δ'. So far as the BZ is concerned, an increase in P_{MU}, σ_{PLS}, or t will reduce BZ, an increase in σ_{MU} or P_{PLS} will increase the BZ, while the effect of an increase in T, MUP, or K on BZ is indeterminate.

4. Conclusion

According to established research, while PLS financing is supposed to be the mainstay of ZIFS, it has been an uphill task to make this form of contract popular. Instead, the normatively weaker, alternative MU contract appears to predominate. The goal of this paper has been to explore and explain the feasibility of PLS contracts undertaken under ZIFS. Two situations were considered: Moving from either MU depositor or MU borrower status to PLS borrower status. The technique used involved establishing a bargaining zone for each case with upper and lower limits designating, respectively, the minimum acceptable share-rate to the bank (ask-rate) and the maximum possible share-rate offer by the firm to the bank (bid-rate). A necessary condition resulted for a PLS contract to be possible so long as the bid-rate exceeds the ask-rate.

(13) See Appendix II.

Our analysis addresses issues extant in the literature relating to: expectations and risk, AS and MH, entrepreneur's share of profit, firm's share of investment funds, the permanent nature of shares to the bank, an external imposition of the profit share rate such as a 50-50 split between the two parties, and related to the rate of return to capital, etc. As to borrowing from the bank, firm's reimbursement schemes used here lend a dual interpretation: paid as a constant sum per period over the duration of the contract or as a lump sum balloon payment at the very end. Through comparative statics analyses for both models, we are able to establish the response of firm's maximum bid-rate, λ', and bank's maximum ask-rate, δ', to changes in various market and non-market parameters. This allows us to predict how BZ may change and how private choices and pubic decisions are likely to affect BZ and the subsequently contracted share-rates. In fact, all of the signs of the first derivatives are heuristically reasonable. Thus, not only do we have testable hypotheses, we also have formulae that may be used empirically to calculate various elasticities of response. This should be very useful to various stakeholders.

Our MUP and cumulative PLS profit formulations are pre-tax constructs. The elements that have not been explored here include: input prices, sectoral wage differentials, subsidies, quotas, tariffs, exchange rate, income tax, etc. on BZ. However, we can see how changes in any of these variables affect the opportunity costs and K, and then using the chain-rule approach we can trace their impact on λ' and δ'.

While we follow Sharpe (1994) in discounting expected profit for the risks faced, we measure risk as a proportional stochastic process. So, we are able to get expected dollars per unit of risk. Hence, opportunity cost and future earnings are made equivalent in terms of weighted units.

There is no upfront inflation adjustment allowance because, on the one hand, the mark-up rate demanded by the bank is presumably inflation adjusted, and on the other, the cumulative profit projected to be earned under a PLS undertaking is viewed as being a stream of future earnings that automatically changes with any change in price – inflation or deflation.

Thus, here, besides establishing a BZ for sharing profit whether among institutions (firm and bank) or factors of production (capital and entrepreneur), we are able to determine optimal capital contributions by the firm under restrictive circumstance. Hence, variations in a bank's (or capital's) contracted share-rate of PLS profit among various firms is well supported by the possible variations in the circumstances of the firms.

Our theoretical research calls for an empirical validation of the results with industry-wide macro-level data. It gives one the opportunity to empirically test them and compare those with existing PLS contracts to identify the level of convergence. In the process one could provide detailed country/sector wide information as to the duration of PLS contracts, size of financing as percentage of the size of total undertakings by the firms, the contracted profit and loss share rates, the rates of return on capital, the industry to which financing were extended, whether the financing were for start-up firms or to firms seeking expansion or replacement equipment, whether the financing covered both fixed capital and operating expenses, the nature or level of bank oversight, depreciation allowance allowed and taken as well as taxes paid by the firms, issues which necessitate conflict resolution, etc.

Further, to test the efficacy of the suggested model herein, one could float this model to banks and firms and observe their responses to it. After all, our research provides banks and firms a heretofore unique and comprehensive rationale for drawing up PLS contract, which happens also to be the optimal recourse. In either case, as a start, one may have to turn to banks in economies where some level of PLS contracts exist in banks' lending portfolios.

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APPENDIX (I)

$\partial\lambda'/\partial m = -[(P_B/\sigma_B)(TfsF)]/[(P_{PLS}/\sigma_{PLS})K] < 0$ (1.3.1)

$\partial\delta'/\partial m = [(P_B/\sigma_B)(1 - f)(1 - s)F\{(T + v)/2\}]/[(P_{PLS}/\sigma_{PLS})K] > 0$ (1.4.1)

$\partial\lambda'/\partial F = -[(P_B/\sigma_B)(Tfms)]/[(P_{PLS}/\sigma_{PLS})K] < 0$ (1.3.2)

$\partial\delta'/\partial F = [(P_B/\sigma_B)m(1 - f)(1 - s)\{(T + v)/2\}]/[(P_{PLS}/\sigma_{PLS})K] > 0$ (1.4.2)

$\partial\lambda'/\partial f = -[(P_B/\sigma_B)(TmsF)]/[(P_{PLS}/\sigma_{PLS})K] < 0$ (1.3.3)

$\partial\delta'/\partial f = -[(P_B/\sigma_B)m(1 - s)F\{(T + v)/2\}]/[(P_{PLS}/\sigma_{PLS})K] < 0$ (1.4.3)

$\partial\lambda'/\partial T = -[(P_B/\sigma_B)(fmsF)]/[(P_{PLS}/\sigma_{PLS})K] < 0$ (1.3.4)

$\partial\delta'/\partial T = [(P_B/\sigma_B)\{m(1 - f)(1 - s)F/2\}]/[(P_{PLS}/\sigma_{PLS})K] > 0$ (1.4.4)

$\partial\lambda'/\partial s = -[(P_B/\sigma_B)(TfmF)]/[(P_{PLS}/\sigma_{PLS})K] < 0$ (1.3.5)

$\partial\delta'/\partial s = -[(P_B/\sigma_B)m(1 - f)F\{(T + v)/2\}]/[(P_{PLS}/\sigma_{PLS})K] < 0$ (1.4.5)

$\partial\lambda'/\partial t = 0$ (1.3.6)

$\partial\delta'/\partial t = 1/K > 0$ (1.4.6)

$\partial\lambda'/\partial E = -(P_a/\sigma_a)/[(P_{PLS}/\sigma_{PLS})K] < 0$ (1.3.7)

$\partial\delta'/\partial E = 0$ (1.4.7)

$\partial\lambda'/\partial K = [(P_B/\sigma_B)TfmsF + (P_a/\sigma_a)E]/(P_{PLS}/\sigma_{PLS})K^2] > 0$ (1.3.8)

$\partial\delta'/\partial K = -[\{(P_B/\sigma_B)m(1-f)(1-s)F\{(T+ v)/2\}\} + (P_{PLS}/\sigma_{PLS})t]/(P_{PLS}/\sigma_{PLS})K^2] < 0$ (1.4.8)

$\partial\lambda'/\partial P_B = -TfmsF/\sigma_{BP}]/[(P_{PLS}/\sigma_{PLS})K] < 0$ (1.3.9)

$\partial\delta'/\partial P_B = [m(1 - f)(1 - s)F\{(T + v)/2\}/\sigma_B]/[(P_{PLS}/\sigma_{PLS})K] > 0$ (1.4.9)

$\partial\lambda'/\partial\sigma_B = [(P_B/\sigma_B^2)TfmsF]/[(P_{PLS}/\sigma_{PLS})K] > 0$ (1.3.10)

$\partial\delta'/\partial\sigma_B =- [(P_B/\sigma_B^2)m(1 - f)(1 - s)F\{(T + v)/2\}/(P_{PLS}/\sigma_{PLS})K] < 0$ (1.4.10)

$\partial\lambda'/\partial P_{PLS} = [(\sigma_{PLS}/\sigma_B)P_BTfmsF + (P_a/\sigma_a)E] /KP_{PLS}^2 > 0$ (1.3.11)

$\partial\delta'/\partial P_{PLS} = -[(P_B/\sigma_B)m(1 - f)(1 - s)F(T + v)/2\}]/[(P_{PLS}^2/\sigma_{PLS})K < 0$ (1.4.11)

$\partial\lambda'/\partial\sigma_{PLS} = -[(P_B/\sigma_B))TfmsF + (P_a/\sigma_a)E] /P_{PLS}K < 0$ (1.3.12)

$\partial\delta'/\partial\sigma_{PLS} = [P_{PLS}m(1 - f)(1 - s)F\{(T + v)/2\}]/P_{PLS}K > 0$ (1.4.12)

$\partial\lambda'/\partial P_a = -(E/\sigma_a)/[(P_{PLS}/\sigma_{PLS})K] < 0$ (1.3.13)

$\partial\delta'/\partial P_a = 0$ (1.4.13)

$\partial\lambda'/\partial\sigma_a = (P_a/\sigma_a^2)E/[(P_{PLS}/\sigma_{PLS})K] > 0$ (1.3.14)

$\partial\delta'/\partial\sigma_a = 0$ (1.4.14)

APPENDIX (II)

$\partial\lambda'/\partial T = [(P_{MU}^F/\sigma_{MU}^F)MUP/2]/[(P_{PLS}/\sigma_{PLS})K > 0$ (2.3.1)

$\partial\delta'/\partial T = [(P_{MU}^B/\sigma_{MU}^B)MUP/2]/[(P_{PLS}/\sigma_{PLS})K] > 0$ (2.4.1)

$\partial\lambda'/\partial t = 0$ (2.3.2)

$\partial\delta'/\partial t = 1/K > 0$ (2.4.2)

$\partial\lambda'/\partial MUP = [(P_{MU}^F/\sigma_{MU}^F)\{(T + v)/2\}]/[(P_{PLS}/\sigma_{PLS})K > 0$ (2.3.3)

$\partial\delta'/\partial MUP = [(P_{MU}^B/\sigma_{MU}^B)\{(T + v)/2\}]/[(P_{PLS}/\sigma_{PLS})K] > 0$ (2.4.3)

$\partial\lambda'/\partial K = [-(P_{MU}^F/\sigma_{MU}^F)\{(T + v)/2\}MUP]/[(P_{PLS}/\sigma_{PLS})K^2 < 0$ (2.3.4)

$\partial\delta'/\partial K = - [(P_{MU}^B/\sigma_{MU}^B)\{(T + v)/2\}MUP + (P_{PLS}/\sigma_{PLS})t]/[(P_{PLS}/\sigma_{PLS})K^2] < 0$ (2.4.4)

$\partial\lambda'/\partial P_{MU}^F = -(1/\sigma_{MU}^F)[K - \{(T + v)/2\}MUP]/[(P_{PLS}/\sigma_{PLS})K < 0$ (2.3.5)

$\partial\delta'/\partial P_{MU}^B = [(1/\sigma_{MU}^B)\{(T + v)/2\}MUP]/[(P_{PLS}/\sigma_{PLS})K] > 0$ (2.4.5)

$\partial\lambda'/\partial P_{PLS} = [(P_{MU}^F/\sigma_{MU}^F)[K - \{(T + v)/2\}MUP]/[(P_{PLS}^2/\sigma_{PLS})K > 0$ (2.3.6)

$\partial\delta'/\partial P_{PLS} = - [(P_{MU}^B/\sigma_{MU}^B)\{(T + v)/2\}MUP]/]/[(P_{PLS}^2/\sigma_{PLS})K < 0$ (2.4.6)

$\partial\lambda'/\partial\sigma_{MU}^F = [(P_{MU}^F/\sigma_{MU}^{F2})[K - \{(T + v)/2\}MUP]/[(P_{PLS}/\sigma_{PLS})K > 0$ (2.3.7)

$\partial\delta'/\partial\sigma_{MU}^B = - [(P_{MU}^B/\sigma_{MU}^{B2})\{(T + v)/2\}MUP]/[(P_{PLS}/\sigma_{PLS})K] < 0$ (2.4.7)

$\partial\lambda'/\partial\sigma_{PLS} = - [(P_{MU}^F/\sigma_{MU}^F)[K - \{(T + v)/2\}MUP]/[P_{PLS}K] < 0$ (2.3.8)

$\partial\delta'/\partial\sigma_{PLS} = [(P_{MU}^B/\sigma_{MU}^B)\{(T + v)/2\}MUP]/[P_{PLS}K] > 0$ (2.4.8)

عقد المشاركة من الأرباح والخسائر
في ظل نظام مالي بدون ربا

شافي خالد وعبدالوهاب خندكر

المستخلص: في ظل نظام مالي خالٍ من الفائدة، يفترض أن يكون عقد المشاركة في الأرباح والخسائر هو العقد الأساسي للقيام بالوساطة المالية، إلا أنه من الناحية العملية الأمر على خلاف ذلك. لقد أضحى عقد المرابحة هو المفضل لدى المؤسسات التمويلية الإسلامية. إن لكلا العقدين (النظري)، والتطبيقي (المرابحة) مزايا وعيوب. فقد أثيرت تساؤلات حول مدى توافق عقد المرابحة المطبق مع أحكام الشريعة الإسلامية ومقاصدها. ومهما قد يبدو من غموض أخلاقي الذي يرافق عمليات المرابحة، فإن الفشل في صياغة عقد مناسب للمشاركة في الربح والخسارة وتطبيقه لم يترك خيارًا سوى الاعتماد على المرابحة. للأسباب المذكورة في الورقة، فإن المرجح أن تهيمن المرابحة على أرض الواقع. باستخدام نموذج تعظيم الشركة لأرباحها، تبحث الورقة رغبة رجل أعمال محايد للخطر للدخول في عقد المشاركة تحت حالتين مختلفتين: الأولى، عندما يكون عقد المشاركة هو الطريق الوحيد للتمويل؛ والثانية، عندما يكون كلاً من عقدي المرابحة، والمشاركة متاحًا كطرق للتمويل. وتناقش الورقة العوامل التي تؤثر على هذا الاختيار.

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