

Capital Asset Pricing Model and Pricing of Islamic Financial Instruments

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Abstract. The prohibitions of *ribā* (interest) and trading in non-*halāl* businesses in Islam redefines investment universe assumed by Capital Asset Pricing Model (CAPM). In Islamic investment world, risk-free asset and market portfolio with non-*halāl* constituents are irrelevant. We propose two-versions of Sharī'ah-compliant CAPM that use Sharī'ah-complaint market portfolio and are free of risk-free asset. We test the proposed models together with conventional CAPM in Bursa Malaysia using three- and ten-year data. The test results of the models with three-year data reveal remarkable similarity between versions of Sharī'ah-compliant models. Besides, Sharī'ah-compliant CAPM is as capable of explaining returns on Sharī'ah-complaint stocks as conventional CAPM is of explaining returns on all stocks.

Keywords: Asset pricing, Sharī'ah-compliant CAPM, Sharī'ah-compliant zero-beta portfolio, Sharī'ah-compliant market portfolio, Islamic financial instruments

JEL Classification: G11, G12, G15

KAUJIE Classification: L31, I41, I43

1. Introduction

Capital asset pricing model (CAPM) of William Sharpe (1964) extends Markowitz's mean variance theory (MVT) by adding the assumptions of the presence of risk-free lending and borrowing and homogeneity of the expectations of investors. CAPM combines capital market line (CML) and Markowitz's efficient frontier to identify a tangency market portfolio that consists of all risky assets in the market. The market portfolio is assumed to be

sufficient for explaining returns on any risk asset. Furthermore, market portfolio together with a risk-free asset, the return on which represents the intercept of CML, constitutes the investment universe for investors. However, CAPM with such an investment universe is not applicable to the explanation of returns on Islamic financial instruments because of the restrictions imposed by Sharī'ah on dealing with interest (*ribā*) and engaging in non-*halāl* businesses.

Allah Almighty admonishes Muslims of indulging in *ribā* by saying:

Those who eat *ribā* will not stand (on the Day of Resurrection) except like the standing of a person beaten by *Shayṭān* (Satan) leading him to insanity. ... So whosoever receives an admonition from his Lord and stops eating *ribā*, shall not be punished for the past; his case is for Allah; but whoever returns (to *ribā*) such are the dwellers of the Fire (Qur'ān 2: 275).

Allah also says:

O you who believe! Be afraid (have *taqwā*) of Allah and give up what remains from *ribā*, if you are (really) believers. And if you do not do it, then take a notice of war from Allah and His Messenger but if you repent, you shall have your capital sums. Deal not unjustly, and you shall not be dealt with unjustly (Qur'ān 2:278-279).

Given the severity of indulgence in *ribā*, Muslim investors are not expected to hold risk-free interest-bearing assets of CAPM. In addition, Allah (SWT) commands Muslims to stay away from forbidden, non-*ḥalāl* businesses, saying:

O you who believe! Indeed intoxicants, gambling, sacrificing on stone alters (to other than Allah), and divining arrows are but defilement from the work of *Shayṭān* (Satan). So avoid that in order that you may be successful (Qur'ān 5:90).

Hence, the market portfolio of CAPM that consists of the stocks of both *ḥalāl* and non-*ḥalāl* businesses is not a permissible investment option for Muslim investors.

The Qur'ānic injunctions redraw the domain of Muslim's investment activities. The prohibition of *ribā* drops out interest-bearing risk-free assets from the investment universe and prohibition of engaging in non-*ḥalāl* businesses necessitates restructuring of the market portfolio. Under these circumstances, the natural questions that arise are: 1) Is CAPM relevant for explaining returns on assets in the Islamic investment universe, 2) If it is, then what needs to be done to make it applicable in Islamic financial

universe. To answer these questions, we propose two alternative versions of CAPM with investment universes that are fully in compliance with the dictates of Sharī'ah. First, we follow Black (1972) and relax the assumption of risk-free lending and borrowing in CAPM and propose use of a Sharī'ah-compliant zero-beta portfolio in place of the risk-free asset. Second, assuming absence of a zero-beta portfolio, we pursue Lintner (1965) approach and replace risk-free asset with cash holding; cash is an asset that does not earn any returns. In addition, we suggest use of a risky market portfolio that is constituted by the stocks of Sharī'ah-compliant businesses only.

The need for a fully Sharī'ah-compliant CAPM is warranted by the phenomenal growth of Islamic financial system in the last three decades. Over years, a number of Sharī'ah-compliant Islamic financial instruments that cater to various investment needs of Muslim investors have been introduced. Because Islamic financial instruments differ from the conventional financial instruments in terms of their legal framework and the nature of transactions, it is imperative that asset pricing models originally designed for conventional financial instruments are customized so that they are applicable to Islamic financial instruments (Jobst, 2007). Furthermore, sustainability of the attractiveness of Islamic financial instruments is contingent on investors' ability to fairly assess their risk return dynamics. Recognizing the need for asset pricing models that not only are based on the investment alternatives permitted by Sharī'ah but also are capable of capturing the true nature of the risk and return dynamics of Islamic financial instruments, we propose two versions of Sharī'ah-compliant CAPM (SCAPM) that are free of the elements forbidden by Sharī'ah. We do acknowledge the shortcomings of CAPM assertion of the sufficiency of market portfolio in explaining stock returns, which in fact, led to the emergence of multifactor asset pricing models. However, the multifactor asset pricing models recognize the dominance of market portfolio in explaining returns of risky assets (Fama and French, 1996; Carhart, 1997; Pastor and Stambaugh, 2003). Hence, we deem it worthwhile to focus on developing CAPM that adheres to the dictates of Sharī'ah and is useful for investors in Islamic financial system.

The proposed models of this study are tested in Bursa Malaysia (Malaysian Stock Market) to estimate returns on nine portfolios constructed from the stocks of the Sharī'ah-compliant firms listed on the Bursa. We use FTSE Bursa Malaysia EMAS Sharī'ah Index as proxy for the Sharī'ah-compliant market portfolio and one-year yield of AAA-rated *ṣukūk* as proxy for the zero-beta portfolio. However, due to limited data available on AAA-*ṣukūk*, we are restricted to a three-year window for testing the SCAPM that replaces risk-free rate with return on Sharī'ah-compliant zero-beta portfolio. To assess performance of this model, we run other models over the same period. The test results of the SCAPMs exhibit remarkable similarity in beta coefficients and explanatory powers to the extent that they seem to be useable interchangeably. Therefore, we test SCAPM with cash and conventional CAPM over a ten-year period from January 2004 to December 2013. We observe that despite differences in the model inputs, the behavior of the Sharī'ah-compliant CAPM is as good an indicator of the returns on Sharī'ah-compliant assets as conventional CAPM is of all risky assets.

The rest of the paper is organized as follows: section 2 reviews the studies related to this study, section 3 proposes CAPM for Islamic financial instruments, section 4 explains data and methodology, section 5 discusses the empirical results, and section 6 concludes the paper.

2. Related Studies

Islamic financial system is expected to be based on the fundamental principle of total submission to the commands of Allah, including adherence to highest moral values and establishment of social justice. The forbiddance of *ribā* bears on ethical and moral dimensions of the Islamic financial system (Mulcahy, 2014). The *ribā*-based system encourages accumulation of wealth in the hands of few individuals, who tend to be disinterested in the welfare of their fellow human beings (Karsten, 1982). Furthermore, *ribā* represents one end of the two forbidden extremes of risk taking, that of avoidance of risk in wealth creation. The other extreme of high risk behavior involves gambling and speculative activities (Zaher and Hassan, 2001; Rosly and Zaini, 2008). Islamic finance recognizes the relationship between risk and return on assets, but

demands observance of high moral values in handling risk and return dynamics (Rosly and Zaini, 2008).

The significant growth of the Islamic financial system in recent decades has led to the emergence of a number of Sharī'ah-compliant Islamic financial instruments customized to the investment needs of Muslim investors. These instruments are structurally different from conventional financial instruments with respect to their legal framework and nature of transactions, which necessitate customization of conventional asset pricing models for pricing them (Jobst, 2007). However, Islamic finance researchers have paid little attention to the pricing of Islamic financial instruments. Moreover, previous studies on Islamic asset pricing have limited scope because of their lack of comprehensiveness in capturing the defining characteristics of Islamic financial instruments.

Tomkins and Karim (1987) suggest removal of risk-free rate from the CAPM equation such that the returns on assets are linearly dependent on their standardized covariance with the market portfolio. The study acknowledges prohibition of *ribā* as well as sufficiency of the market portfolio in explaining asset returns. However, it ignores prohibition of dealing with non-*ḥalāl* businesses that are components of the market portfolio. Several other studies propose use of different alternatives of risk-free rate, such as nominal GDP (Shaikh, 2009), *zakāh* rate (El-Ashker, 1987), and inflation (Hanif, 2011). These studies fall short of suggesting a SCAPM that does not violate the fundamental principles of CAPM theory. Shaikh (2009) uses nominal GDP lagged by one year, which is not in synch with single-period static CAPM that assumes presence of instantaneous risk-free assets. An instantaneous risk-free is an asset about which at any instant of time investors are certain about its expected returns (Merton, 1973). Furthermore, CAPM is an extension of Markowitz's mean variance theory (MVT), which states that investors can diversify risk of their portfolios by combining risky assets that are not perfectly correlated with one another. CAPM extends MVT by combining a risky investment in the form of market portfolio with a risk-free asset. In other words, the inputs to CAPM are returns that investors can earn on the investment alternatives. To the best knowledge of the researchers, there is not any asset that earns nominal GDP, *zakāh* rate, or inflation rate. In addition, inflation rate and GDP are expected

to affect cross section of returns in a market and were used as state variables in the first economic multifactor model by Chen, Roll, and Ross (1986). The state variables cannot replace risk-free asset or zero beta portfolio, which are insulated from the fluctuations in the economy (Black, 1972; Wei, 1988; Fama, 1996). Moreover, the use of conventional market portfolio in these studies ignores restrictions imposed by Shari'ah on dealing with non-*halal* businesses, which is tantamount to treating Islamic financial market a subset of the overall market, to which we disagree. We treat an Islamic financial market as a separate entity that is independent of the conventional market.

Several other studies have used CAPM and other conventional performance measurement tools to evaluate the performance of Islamic financial instruments. Selim (2008) illustrates that within CAPM framework, zero risk-free rate optimizes the risk-return relationship in *musharakah* contracts. In his derivation, he uses total return that includes both economic profit and opportunity cost of the invested capital, computed as the risk-free return. Abbas (2012) ignores prohibition of *riba* and applies conventional CAPM and Sharpe ratio to compare rate of return on Islamic and conventional market indices. Dewi and Ferdian (2012) follow similar procedures in evaluating performance of Islamic mutual funds in Malaysia and Indonesia. Similarly, Affaneh, Albohali, and Boldin (2013) compare performance of the Islamic and conventional mutual funds using Sharpe ratio adjusted for autocorrelation and skewness with excess portfolio returns computed net of risk-free rate.

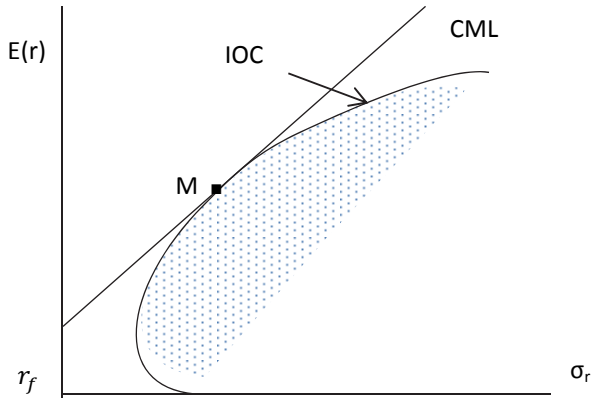
All these studies overlook the structural differences between conventional and Islamic financial products caused by the restrictions imposed by the Shari'ah. They treat Islamic instruments as subset of the overall market, the constituents of which can be evaluated like any other asset in the market. Recognizing the uniqueness of the Islamic instruments and the investment universe they belong to, this study proposes versions of SCAPM that use inputs that are in total compliance with the dictates of Shari'ah and are free of forbidden elements of *riba* and non-*halal* businesses. Besides, it upholds the fundamental principles of CAPM, those of returns used in the model are returns or lack of returns on the assets held or invested in.

3. CAPM for the Islamic Financial Instruments

Sharpe (1964) extends Markowitz's mean variance theory (MVT) to develop a capital asset pricing model (CAPM) under the condition of market equilibrium. He adds two assumptions to MVT⁽¹⁾: (i) investors can lend and borrow at pure risk-free rate, and (ii) investors have homogenous expectations and evaluate the available investment options identically. He combines capital market line (CML) and Markowitz's efficient frontier to define the investment opportunity set for the investors. The intercept of CML represents return on risk-free asset and CML's point of tangency on efficient frontier represents a risky portfolio that is constituted by all risky assets (see figure 1 below). He concludes that risk-free asset and risky portfolio represent investment universe under the condition of market equilibrium and tangency portfolio is sufficient to explain returns on any risky asset (for derivation of CAPM, refer to section I in Appendix).

(1) Assumptions of MVT: 1) Investors characterize investments based on probability distribution of their expected holding period returns. 2) The risk of an investment is reflected by the variance of its returns. 3) Investors make decisions on the basis of probability distribution of the expected returns and their volatility. 4) Investors are risk averse and prefer higher return for a given level of risk and lower risk for a given level of return. 5) Investors aim at maximizing their one-period expected utility (Reilly and Brown, 2003).

Figure 1: The Capital Market on the Efficient Frontier



Source: Sharpe (1964), *Journal of Finance*, XIX (3) p. 436. [Axes switched by authors]

The CAPM of Sharpe cannot be used directly for estimating returns of risky Islamic financial instruments because the inputs of the model do not belong to the investment universe of Muslim investors. Islam strictly prohibits investing in interest-bearing risk-free assets and non-*halāl* businesses. Therefore, it is necessary to customize CAPM according to the commands of Shari‘ah in order to make it applicable to the Islamic financial instruments. In this direction, we adopt two approaches to propose two versions of Shari‘ah-complaint CAPM (SCAPM).

First, we appeal to Black (1972) version of CAPM that relaxes the assumption of risk-free lending and borrowing in order to explain the deviations of actual returns of low beta and high beta firms, which were consistency higher and lower than the returns estimated by CAPM. Black contends that combining a zero-beta portfolio with the market portfolio in CAPM explains the discrepancies in actual returns and CAPM estimated returns. He illustrates that all efficient portfolios, including market and zero-beta portfolios, can be created from two basic portfolios with different betas (For details, refer to section II in Appendix). Black (1972) model is of the form:

$$E(r_i) = \beta_i E(r_M) + (1 - \beta_i) E(r_z),$$

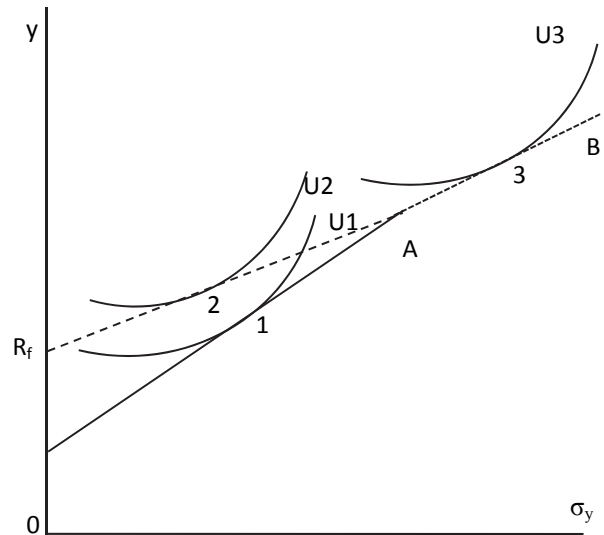
which after rearrangement gives

$$E(r_i) = E(r_z) + \beta_i [E(r_M) - E(r_z)] \quad (1)$$

where, r_i , r_z , and r_M are returns of asset i , zero-beta portfolio, and market portfolio respectively, and β_i is the systematic risk of i . $E(\cdot)$ is the expectation factor.

Second, in case zero-beta portfolio is not available, we suggest removal of risk-free rate from the CAPM. This approach we adopt from Lintner (1965) who, while relaxing the assumption of homogeneity of investors in CAPM, suggests several investment combinations that investors can avail of. Of those alternatives, we use Case I, which says that investors have the option of holding cash and investing in a risky asset (for details, refer to section III in Appendix). An investor can change his overall return and risk as he may desire along the market opportunity line (see figure 2), which is the equivalent of CML in CAPM.

Figure 2: Investment Choices Involving A Single Stock



Source: Lintner (1965), *Journal of Finance*, XX(4), p. 590.

Point A: All investment in i .

Case 1-0A: Market opportunity line between i and cash.

Case 2- R_f A: Market opportunity line between i and saving deposit paying R_f

Case 3- R_f AB: Opportunity to borrow at R_f

Using market portfolio as the risky asset, the Lintner (1965) model modifies as

$$E(r_i) = \beta_i [E(r_M)] \quad (2)$$

Models (1) and (2) address the prohibition of *ribā*. Besides, the prohibition of investing in non-*halāl* businesses is overcome by recognizing Islamic

financial markets are independent entities, wherein only Shari'ah-complaint assets are traded. In such markets, the market portfolio is made up of Shari'ah-complaint stocks only. Combining (1) and (2) with Shari'ah-compliant market portfolio, we propose the following two versions of Shari'ah-compliant CAPM (SCAPM).

1. SCAPM with zero-beta portfolio [SCAPM(Z)]⁽²⁾

$$E(r_{i_s}) = E(r_{z_s}) + \beta_{i_s}[E(r_{M_s}) - E(r_{z_s})] \quad (3)$$

2. SCAPM with cash or zero risk-free rate [SCAPM(C)]

$$E(r_{i_s}) = \beta_{i_s}[E(r_{M_s})] \quad (4)$$

where, r_{i_s} , r_{z_s} and r_{M_s} are returns on the Shari'ah-compliant stock, zero-beta portfolio and market portfolio respectively. β_{i_s} is the standardized covariance between return on Shari'ah-compliant stock i_s and return on Shari'ah-compliant market portfolio M_s .

Taking off expectations and rearranging transforms (3) and (4) into

$$r_{i_s} - r_{z_s} = \alpha_{i_s} + \beta_{i_s}[r_{M_s} - r_{z_s}] + \varepsilon_{i_s} \quad (5)$$

and

$$r_{i_s} = \alpha_{i_s} + \beta_{i_s}(r_{M_s}) + \varepsilon_{i_s} \quad (6)$$

where, α_{i_s} and ε_{i_s} are the intercept and error terms respectively. The equations (5) and (6), in fact, represent the models, SCAPM, of this study. In addition, we test conventional CAPM (CAPM)

$$r_i - r_f = \alpha_i + \beta_i[r_M - r_f] + \varepsilon_i \quad (7)$$

where, r_i r_f r_M are returns on asset i , risk-free asset, and market portfolio respectively.

4. Data and Methodology

The proposed models of this study are tested in Bursa Malaysia (Malaysian Stock Exchange) using data of Shari'ah-compliant non-financial firms listed on the Bursa. Alongside, we test the CAPM using all non-financial firms. The firms are included in the study if they meet two criteria: 1) they have data available for at least two years, and 2) they do not carry negative equity (Fama and French, 1993). We screened firms at the beginning of each year to check their equity position using end of the preceding year's data. The number of firms that met the set criteria in each year is given in Table 1:

For risk-free rate, we use return on 3-month Malaysian Treasury bill and for return on zero-beta portfolio, we use one year yield on AAA-*şukūk*. The data on the returns on the stock of firms and firm-related variables, risk-free rate and AAA-*şukūk* are obtained from the Bloomberg Professional Database.

Due to discrepancy in the data available on the model inputs, we run the models over two periods. The data on all model inputs except AAA-*şukūk*, the proxy for Shari'ah-compliant zero-beta portfolio, is available for ten year period. The earliest available data point for AAA-*şukūk* is November, 2011, and we are restricted to three-year period of January 2012 to December 2014 for testing SCAPM(Z). We run other models, SCAPM(C) cash and conventional CAPM, over the same three-year period to assess the comparative performance of the models. We acknowledge that three-year data is not enough to draw reliable conclusions on the behavior of models.

Table 1: Number of Firms Included in the Study

Year	2014	2013	2012	2011	2010	2009	2008	2007	2006	2005	2004
SC	469	479	481	472	458	449	436	417	403	208	208
SC+NC	735	754	755	733	709	695	672	647	614	536	300

SC: Shariah-compliant; NC: Non-Shariah compliant

(2) We are thankful to our reviewers for suggesting this version of CAPM; it uses AAA-*şukūk* as proxy for zero-beta portfolio.

However, the test results are expected to provide a firsthand feel of the behavior of the model parameters. In addition, we test SCAPM(C) and CAPM over ten-year period of January 2004 to December 2013.

4.1 Model Inputs

Explanatory variables

CAPM is a single-factor that assumes sufficiency of market portfolio in explaining returns on risky stocks. In SCAPM, we use FTSE Bursa Malaysia EMAS Shari'ah Index as proxy for the market portfolio. The index is constituted by Shari'ah-compliant firms in the FTSE Bursa Malaysia EMAS Index that meet the screening requirement of the Shari'ah Advisory Council (SAC)⁽³⁾. As explanatory variables, we use returns on the index in excess of one-year yield on AAA-*ṣukūk* in SCAPM(Z), and actual returns on the index in SCAPM(C).

We test the conventional CAPM with return on FTSE Bursa Malaysia EMAS Index, in excess of return on 3-month Malaysian Treasury bill, as explanatory variable. FTSE Bursa Malaysia EMAS Index is a market capitalization-weighted index that consists of large cap and mid cap constituents of the FTSE Bursa Malaysia 100 Index and FTSE Bursa Malaysia Small Cap Index (FTSE Bursa Malaysia Index Series, 2014).

Dependent variables

The dependent variables in the models comprise of returns on nine portfolios created from the selected firms. We apply Fama and French (1993) sorting procedure, which involves sorting firms first on size then on book to market value of equity (B/M). In first sort, firms are divided in the proportion of 33:34:33 into three size groups: small (S), medium (M), and big (B). In second sort, the firms in each size group are further divided to create

three B/M groups in the proportion of 33:34:33. The three groups thus obtained are high B/M (H), medium B/M (M) and low B/M (L). From the intersection of this 3x3 sort, nine portfolios, namely small size high B/M (SH), small size medium B/M (SM), small size low B/M (SL), medium size high B/M (MH), medium size medium B/M (MM), medium size low B/M (ML), big size high B/M (BH), big size medium B/M (BM), and big size low B/M (BL) are created. We use value-weighted returns on these portfolios as dependent variables.

The dependent portfolios for SCAPM are created from the Shari'ah-compliant firms listed on the Bursa Malaysia (Bursa Malaysia, List of Companies, 2014) while portfolios for CAPM are created from all listed firms. The return on dependent portfolios for SCAPM(Z) is computed net of yield on AAA *ṣukūk* and the return on dependent portfolios for CAPM is computed net of rerun on 3-month Malaysian Treasury security. In case of SCAPM(C), actual return on portfolios is used. The dependent portfolios in all models are revised annually.

4.2 Descriptive Statistics of the Model Inputs

As mentioned earlier, we test our models over two periods: all models are tested over a three-year period, and SCAPM(C) and CAPM are also tested over a ten-year period. Therefore, we present the descriptive statistics of the three-year data first followed by the descriptives of the ten-year data.

4.2.1 Three Year Data

Descriptive Statistics of SCAPM(Z) Inputs

The mean monthly excess return on Shari'ah-compliant market portfolio is 0.53 percent with standard deviation of 2.50 percent per month. The mean excess returns on dependent variables range from 0.40 (BM) to 2.00 (SH); their standard deviations range from 2.26 percent (BL) to 4.67 percent (SH). The returns on portfolios display size effect. Controlling for B/M, the returns fall monotonically from small size portfolios to big size portfolios. However, the value effect is not seen within size groups. The Jarque-Bera (JB) statistics do not reject the null of normal distribution in data for SL, BM, and BL portfolios; in remaining portfolios, it is rejected at 1 percent significance level. Still, data distribution for all

(3) SAC requirements, as highlighted in the fact sheet of FTSE Bursa Malaysia Index Screening (2014), stipulate that a Shari'ah compliant firm should not be involved in any of the following as core business activity: 1) *ribā*-based financial services, 2) Gambling, 3) Manufacturing or selling of non-*ḥalāl* products, 4) Conventional insurance, 5) Non-permissible entertainment activities, 6) Manufacturing or selling of tobacco or tobacco-related products, 7) Trading in Shari'ah non-compliant securities, and 8) Other activities deemed non-permissible according to Shari'ah.

JB statistics reject null hypothesis of normal distribution for all variables at 1 percent level of significance. The data distribution for all dependent variables is leptokurtic with kurtosis of more than 3; its values range from 3.70 (SM) to 8.67 (SH). The return on the market portfolio has kurtosis of 5.29. Skewness of more than $|1|$ is seen only in the distributions of SH (1.81) and BL (-1.49). For details, see Table 3 panel A.

Descriptive Statistics of CAPM Inputs

The average excess monthly return on the explanatory variable in Conventional CAPM, the market portfolio, is 1.11 percent with standard deviation of 11.06 percent. The excess monthly returns on dependent portfolios are between -0.34 percent (ML) and 1.67 percent (SH), with standard deviations in the range of 3.66 percent (BL) to 5.75 percent (SH). The size and value effects in portfolios are similar to those in SCAPM. JB test statistics lead to the rejection of null hypothesis of normal distribution at 1 percent significance level for all model inputs except SM. The condition of normal distribution is violated by the presence of excess kurtosis in all data series. The kurtosis falls in the range of 3.18 (SM) to 6.95 (SH); the kurtosis of the market portfolio is 5.53. Skewness of more than 1 is seen in SH (1.53) and BL (-1.19) only. For details, see Table 3 panel B.

4.3 Model Testing

We use generalized method of moments (GMM) of Hansen (1982) for data analysis and model testing.

The GMM allows testing of mean-variance efficiency of asset pricing models without need for strong distributional assumption of normality in data distribution and offers a heteroskedasticity and autocorrelation consistent estimation method (Jagannathan, Skoulakis, and Wang, 2002; Verbeek, 2004). Nonetheless, the data series are supposed to be stationary and ergodic time series with finite fourth moment (Jagannathan, Skoulakis, and Wang, 2002). A stochastic data time series is said to be stationary if its variance and covariance are finite and independent of time. An ergodic process permits interchange of time series average with expectations (Rachev, et. al., 2007). It is an important element of estimation theory because its presence along with stationarity ensures that time series estimation parameters are meaningful (Greene, 2011).

We also test each data series for stationarity using augmented Dickey-Fuller (ADF), Philips-Perron (PP), and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests. All stationarity tests show that all variables, model inputs, in both SCAPM and conventional CAPM are stationary at level. Table 3A and Table 3B present unit root test results of inputs to the Shariah compliant and conventional CAPM respectively. In addition, descriptive statistics reveal that fourth moment or kurtosis of the data distribution of all model inputs is finite.

Table 4: Test Results of the Stationarity of the Model Inputs

A. Shariah-Complaint CAPM											
		SH	SM	SL	MH	MM	ML	BH	BM	BL	r_{MS}
ADF	t-Stat	-8.68	-10.23	-9.04	-9.53	-10.07	-10.36	-10.27	-10.48	-5.81	-8.85
	Prob.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PP	Adj. t	-8.81	-10.28	-9.06	-9.78	-10.17	-10.38	-10.33	-10.56	-9.64	-9.07
	Prob.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
KPSS	LM Stat	0.07	0.05	0.05	0.04	0.05	0.04	0.05	0.05	0.06	0.04

B. Conventional CAPM											
		SH	SM	SL	MH	MM	ML	BH	BM	BL	r_M
ADF	t-Stat	-8.98	-9.95	-10.15	-10.11	-4.66	-10.30	-9.14	-9.75	-5.63	-15.05
	Prob.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PP	Adj. t	-9.19	-9.99	-10.20	-10.21	-9.41	-10.32	-9.29	-9.86	-10.00	-15.71
	Prob.	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
KPSS	LM Stat	0.08	0.04	0.05	0.04	0.04	0.06	0.06	0.05	0.06	0.11

5. Empirical Results

This section discusses test results of the proposed SCAPM along with conventional CAPM in Bursa Malaysia. The test results of the models are presented first with three-year data followed by the results of models with ten year data.

5.1 Three Year Data

SCAPM(Z)

The test results of SCAPM(Z) (see Table 5, panel A) reveal that the null hypothesis of zero coefficient ($H_0: \beta_{is}=0$) for Sharī'ah-compliant market portfolio is rejected at 1 percent significance level in all portfolios. The coefficients of the market portfolio range from 0.8483 (BL) to 1.2681 (SH). It is smaller in big size portfolios in which it has values of 0.9082 (BH), 0.9509 (BM), and 0.8483 (BL). The beta coefficients, as measure of systematic risk, exhibit anticipated size effect. Controlling for B/M, the

coefficients fall monotonically from small size portfolio to big size portfolio in all B/M groups. However, after controlling for size, the value effect is not seen in size groups. In small and medium size groups, medium B/M has lowest coefficients and in big size group, medium B/M portfolio has highest coefficient.

The explanatory power of the model, represented by its adjusted R^2 , ranges from 0.3440 (BH) to 0.8690 (BL). In explanatory power of the model, size effect is visible. Controlling for value of B/M, there is monotonic decrease in the adjusted R^2 of the model in high B/M group and monotonic increase in adjusted R^2 in medium B/M group. In low B/M group, size effect does not exist. The expected size effect is displayed by the medium B/M group only. On the other hand, controlling for size, monotonic increase in adjusted R^2 is observed only in big size portfolios.

Table 5: GMM Estimates of the Models
(Three-year Data)

Portfolios		A. SCAPM with Zero Beta Portfolio			B. SCAPM with Cash			C. Conventional CAPM		
		$r_{is} - r_{zs} = \alpha_{is} + \beta_{is}[r_{Ms} - r_{zs}] + \varepsilon_{is}$			$r_{is} = \alpha_{is} + \beta_{is}r_{MS} + \varepsilon_{is}$			$r_i - r_f = \alpha_i + \beta_i[r_M - r_f] + \varepsilon_i$		
		c	Lmk	Adj R ²	c	lmk	Adj R ²	c	lmk	Adj R ²
SH	Coef.	1.3258	1.2681	0.4427	1.2486	1.2690	0.4426	1.1001	1.3646	0.3071
	t-stat	2.06**	3.58***		1.93*	3.58***		1.86*	3.70***	
SM	Coef.	1.0247	1.0526	0.3764	1.0089	1.0533	0.3763	1.3770	0.7724	0.1450
	t-stat	1.70*	3.43***		1.61	3.43***		3.10***	2.61***	
SL	Coef.	0.3426	1.1472	0.4254	0.3001	1.1480	0.4254	1.2590	1.0684	0.1332
	t-stat	0.53	4.07***		0.45	4.08***		1.47	3.27***	
MH	Coef.	0.2030	1.1704	0.4271	0.1543	1.1705	0.4267	1.0406	0.8619	0.1280
	t-stat	0.44	3.40***		0.32	3.09***		1.44	2.14**	
MM	Coef.	0.5735	1.0027	0.4439	0.5729	1.0025	0.4434	1.4622	0.8146	0.1502
	t-stat	1.21	3.64***		1.13	3.63***		2.53***	2.78***	
ML	Coef.	0.4159	1.1232	0.3936	0.3803	1.1239	0.3934	0.9495	0.9396	0.1450
	t-stat	0.72	3.07***		0.64	3.07***		1.38	2.30**	
BH	Coef.	0.0760	0.9082	0.3440	0.1028	0.9074	0.3431	0.8247	0.6782	0.1394
	t-stat	0.19	3.90***		0.24	3.90***		1.09	1.94*	
BM	Coef.	-0.1003	0.9509	0.6652	-0.0863	0.9509	0.6648	0.9881	0.5807	0.1781
	t-stat	-0.35	6.46***		-0.30	6.45***		1.74*	2.54***	
BL	Coef.	0.1871	0.8483	0.8690	0.2304	0.8483	0.8687	1.0613	0.6084	0.1582
	t-stat	1.35	7.99***		1.46	7.97***		2.08**	2.32**	

*** Significant at 1%, ** Significant at 5%, * significant at 10%

The intercept of the model ranges from -0.1003 (BM) to 1.325 (SH). It is significant in SH and SM at 5 percent and 10 percent levels of significance respectively.

SCAPM(C)

The test results of SCAPM(C) (see Table 5, panel B) show that the Shari'ah-compliant market portfolio is significant at 1 percent significance level in all portfolios. Its coefficients range from 0.8483 (BL) to 1.2690 (SH). The coefficients are almost same as those in CAPM with zero-beta portfolio; the maximum difference observed is of 0.0009 in SH portfolio. In addition, the coefficients display size and value effects that are similar to those in SCAPM with zero-beta portfolio. The explanatory power of the model ranges from 0.3431(BH) to 0.8687 (BL); it is similar to that of the preceding model with maximum difference of 0.09 percent in BH. Moreover, the model exhibits size and value effects similar to those of SCAPM with zero-beta portfolio.

The intercept of the model ranges from -0.0863 (BM) to 1.2486 (SH). It is significant in SH at 10 percent significance level.

CAPM

The market portfolio is significant in all portfolios. It is significant at 10 percent in BH, at 5 percent in MH, ML, and BL, and at 1 percent in the remaining five portfolios. Its coefficients range from 0.5807 (BM) to 1.3646 (SH). Controlling for B/M, the coefficients display expected size effect in high and low B/M portfolios only. In these groups, the coefficients fall monotonically from small to big size portfolios. In medium B/M portfolio, medium size portfolio carries highest coefficient. Whereas, controlling for size, we do not observe desired coefficient on different B/M portfolios. In small and big size groups, medium B/M portfolios have lowest coefficient, while in medium size group, low B/M portfolio has the highest coefficient.

The explanatory power of CAPM is much lower than that of its Shari'ah-complaint counterparts. Its adjusted R^2 ranges from 0.1280 (MH) to 0.3071 (SH). It is less than 20 percent in all portfolios except SH. The model displays size effect in medium and low B/M groups, where adjusted R^2 increase monotonically, by small margins though.

Table 6: GMM Estimates of the Models
(Ten-year Data)

Portfolios		A. Shariah-Complaint CAPM with Cash			B. Conventional CAPM		
		$r_{is} = \alpha_{is} + \beta_{is}r_{MS} + \varepsilon_{is}$			$r_i - r_f = \alpha_i + \beta_i(r_M - r_f) + \varepsilon_i$		
		A	r_{MS}	Adj. R^2	α	$r_M - r_f$	Adj. R^2
SH	coef.	1.3071	0.8414	0.2608	1.0216	0.8366	0.3540
	t-Stats	2.45**	5.41***		2.28**	7.00***	
SM	coef.	0.2153	0.7707	0.4545	0.2288	0.6295	0.3668
	t-Stats	0.64	8.04***		0.73	8.96***	
SL	coef.	0.0711	0.6329	0.3544	-0.1601	0.7918	0.3426
	t-Stats	0.20	8.14***		-0.36	7.32***	
MH	coef.	0.2742	1.0035	0.5590	0.0514	0.9534	0.6005
	t-Stats	0.79	9.35***		0.17	10.44***	
MM	coef.	-0.0402	0.8045	0.5999	0.0024	0.7934	0.6215
	t-Stats	-0.165	14.82***		0.01	17.12***	
ML	coef.	-0.4701	0.7122	0.4395	-0.9056	0.7320	0.4898
	t-Stats	-1.63	9.12***		-3.03	10.36***	
BH	coef.	0.3806	1.1563	0.6945	0.3526	1.2023	0.7459
	t-Stats	1.53	19.52***		1.43	18.25***	
BM	coef.	0.4586	0.9204	0.7592	0.2399	0.9362	0.8290
	t-Stats	1.83*	10.81***		1.37	16.69***	
BL	coef.	-0.0252	0.8813	0.8285	-0.1074	0.8079	0.8280
	t-Stats	-0.134	10.78***		-0.68	12.90***	

*** Significant at 1%, ** Significant at 5%, * significant at 10%

The intercept of the model ranges from 0.08247 (BH) to 1.4622 (MM). It is significant in SH and BM at 10 percent, in BL at 5 percent, and in SM and MM at 1 percent significance level. (For details, see Table 5, panel C)

5.2 Ten Year Data

SCAPM(C)

The test results of the SCAPM(C) over ten-year period (see Table 6, panel A) show that the Sharī'ah-compliant market portfolio is significant in all portfolios. The null hypothesis of $\beta_{is} = 0$ is rejected in all portfolios at 1 percent significance level. The beta coefficients of the market portfolio range from 0.63 (SL) to 1.16 (BH). Both value and size effects are visible in the coefficients. Controlling for size, the coefficients decrease monotonically from high B/M portfolios to low B/M portfolios, and controlling for B/M, the coefficients increase monotonically from small size portfolios to big size portfolios. Similar size effect was observed in some previous CAPM studies in developed as well as developing markets (Fama and French, 1993; 1996; Carhart, 1997; Garcia and Ghysels, 1998; Hakim, Hamid, and Meera, 2015).

In all models, size effect is mostly observed in the explanatory power of the models. However, the size effect is not in congruence with the findings of the previous CAPM studies, which report incremental increase in the explanatory power from small size to big size groups, with significantly higher explanatory power in big size portfolios (Fama and French, 1993; Carhart, 1997; Garcia and Ghysels, 1998; Hakim, Hamid, and Meera, 2015).

Still, the explanatory power of SCAPM models is closer to expectations as it is substantially higher in BM and BL portfolios.

We acknowledge that conclusive inference about the behaviors of the models cannot be drawn based on test results from three-year data, to which we were restricted by the limited data availability on AAA-*ṣukūk*. Therefore, we rerun SCAPM (C) and conventional CAPM with available ten-year data.

Over ten year period, the behavior of SCAPM(C) in explaining portfolio returns is similar to that of the conventional CAPM. The coefficients of the market portfolio in SCAPM are close to those in CAPM, with maximum differences of 14 and 16 basis points observed in SM and SL portfolios respectively. In other portfolio, the difference is of 7 basis points or less. In both models, the market portfolio contributes strongly to the explanation of big size portfolios; the finding is in agreement with the findings of the previous studies (Fama and French, 1993; Carhart, 1997; Garcia and Ghysels, 1998; Hakim, Hamid, and Meera, 2015). However, explanatory power of SCAPM is lower in six of the nine portfolios, while it is higher in two small portfolios. The small-firm size effect is seen in the explanatory power of both models. In addition, the intercepts of SCAPM and CAPM are significantly different from zero in two portfolios each. It implies that the interest rate free SCAPM is as good a measure of explaining returns on Sharī'ah-compliant stocks as the conventional CAPM is in explaining returns on stocks in general.

Table 7: Adjusted R² of the Models
(Three-year Data)

	SCAPM(Z)	SCAPM(C)	CAPM	Difference*
SH	0.4427	0.4426	0.3071	0.1354
SM	0.3764	0.3763	0.1450	0.2313
SL	0.4254	0.4254	0.1332	0.2922
MH	0.4271	0.4267	0.1280	0.2987
MM	0.4439	0.4434	0.1502	0.2932
ML	0.3936	0.3934	0.1450	0.2485
BH	0.3440	0.3431	0.1394	0.2038
BM	0.6652	0.6648	0.1781	0.4867
BL	0.8690	0.8687	0.1582	0.7105

* Difference = Min[SCAPM(Z), SCAPM(C)] - CAPM

6. Conclusions

Islam strictly forbids Muslims from paying or receiving interest (*ribā*) and engaging in non-*halāl* businesses, which, for the Sharī'ah-abiding Muslims, redefine the investment universe assumed by CAPM. This, along with uniqueness of Islamic financial instruments, demands customization of conventional CAPM for pricing Islamic financial instruments. In this regard, we propose two versions of Sharī'ah-compliant CAPM. One replaces risk-free rate with Sharī'ah-complaint zero-beta portfolio in the form of AAA-*ṣukūk* and the other drops risk-free rate from the model assuming absence of zero-beta portfolio. In addition, Sharī'ah-complaint market portfolio is used as proxy for the market portfolio.

We test the proposed models together with conventional CAPM in Bursa Malaysia using three-year and ten-year data. We test all models with three year data and SCAPM(C) and conventional CAPM with both three- and ten-year data. The two stage testing is necessitated by limited data availability on AAA-*ṣukūk*. We use nine portfolios each, created from Sharī'ah-compliant and all firm listed on Bursa that meet screening criteria, as dependent variables in SCAPM and CAPM respectively. The test results of the models over three year horizon, show marked similarity between SCAPMs and their distinctive difference from CAPM. However, in ten-year period, the distinction between SCAPM and CAPM fades away. In fact, the models behave closely in explaining portfolio returns.

The findings of this study lead to two conclusions: First, the two versions of Sharī'ah-compliant CAPM yield similar results and have almost same explanatory power in different portfolios. Hence, they can be safely used interchangeably. However, the findings based on three-year data should be handled with care. The similarity of the models needs to be tested over longer time horizon, long enough to make inferences reliable. Second, over longer period, both Sharī'ah-compliant and conventional models display similar behavior in explaining portfolio returns. The similarity in the behaviors exists despite differences in the model inputs, which indicates that Sharī'ah-compatible CAPM is as applicable in explaining risk-return dynamics of the Sharī'ah-compliant stocks as conventional CAPM is in stocks in general. However, it is necessary that the models are tested in other markets to establish their validity.

This study is significant as it proposes asset pricing models that are free of the inputs—interest rate (*ribā*), market portfolio containing non-*halāl* stocks—that are not permissible for Muslim investors. The underlying principle of the models is that the interest rate is not an opportunity cost for the Muslim investors because it is not an investment option or a foregone investment opportunity for them. Moreover, treating Islamic financial markets as independent entities and upholding CAPM theory, the proposed models derive inputs from actual investment universe of Sharī'ah-abiding Muslim investors.

References

- Abbes, M.B.** (2012) Risk and Return of Islamic and Conventional Indices. *International Journal of Euro-Mediterranean Studies*, **5**(1), 1-23.
- Affaneh, I., Albohali, M. and Boldin, R.J.** (2013) Islamic Market Indices: A comparison with conventional world indices based on the adjusted Sharpe ratio. *International Journal of Business, Accounting, and Finance*, **7**(1), 87-100.
- Black, F.** (1972) Capital market equilibrium with restricted borrowing. *Journal of Business*, **45**(3), 444-455.
- Bursa Malaysia, List of Companies** (2014) Retrieved August 20, 2014, from Bursa Malaysia: <http://www.bursamalaysia.com/market/listed-companies/list-of-companies/main-market/#/?alphabet=All>
- Carhart, M.M.** (1997) On persistence in mutual fund performance. *Journal of Finance*, **LII**(1), 57-82.
- Chen, N.-F., Roll, R. and Ross, S.A.** (1986) Economic forces and the stock market. *Journal of Business*, **59**(3), 383-403.
- Cochrane, J.H.** (2001) *Asset Pricing* (1st ed.). Princeton, NJ: Princeton University Press.
- Dewi, M.K. and Ferdian, I.R.** (2012) Evaluating Performance of Islamic Mutual Funds in Indonesia and Malaysia. *Journal of Applied Economics and Business Research*, **2**(1), 11-33.
- El-Ashker, A.A.-F.** (1987) *The Islamic Business Enterprise*. London: Croom Helm.
- Fama, E.F.** (1996) Multifactor portfolio efficiency and multifactor asset pricing. *Journal of Finance and Quantitative Analysis*, **31**(4), 441-465.
- Fama, E.F. and French, K.R.** (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, **33**, 3-56.
- Fama, E.F. and French, K.R.** (1996) Multifactor explanations of asset pricing anomalies. *Journal of Finance*, **LI**(1), 55-84.
- FTSE Bursa Malaysia Index Series** (2014) Retrieved August 25, 2014, from FTSE: <http://www.ftse.com/products/indices/bursa-malaysia>
- Garcia, R. and Ghysels, E.** (1998) Structural change and asset pricing in emerging markets. *Journal of International Money and Finance*, **17**, 455-473.
- Greene, W.** (2011) *Econometric Analysis, 7th Edition*. New Jersey: Prentice Hall.
- Hakim, S., Hamid, Z. and Meera, A.K.** (2015) Combining Local and Global Markets in Asset Pricing in Emerging Markets: Evidence from Three BRICS Nations. *Journal of Developing Area*, **49**(3), 365-378.
- Hanif, M.** (2011) Risk and Return under Shari'a Framework: An Attempt to Develop Shari'a Compliant Asset Pricing Model (SCAPM). *Pakistan Journal of Commerce and Social Sciences*, **5**(2), 283-292.
- Ibn-Kathir**, (2003) *Tafsir Ibn Kathir*. Riyadh: Darassalam.
- Jagannathan, R., Skoulakis, G. and Wang, Z.** (2002) Generalized Method of Moments: Applications in Finance. *Journal of Business & Economic Statistics*, **20**(4), 470-481.
- Jobst, A.A.** (2007) The Economics of Islamic Finance and Securitization. *The Journal of Structured Finance*, **13**(1), 6-27.
- Karsten, I.** (1982) Islam and financial intermediation. *Staff Papers International Monetary Fund*, **29**(1), 108-142.
- Lintner, J.** (1965) Security prices, risk, and maximal gains from diversification. *Journal of Finance*, **XX**(4), 587-615.
- Merton, R.C.** (1973) An intertemporal capital asset pricing model. *Econometrica*, **41**(5), 867-887.
- Mulcahy, M.B.** (2014) Purifying Islamic equities: the interest tax shield. *International Journal of Islamic and Middle Eastern Finance Management*, **7**(4), 473-484.
- Pastor, L. and Stambaugh, R.F.** (2003) Liquidity Risk and Expected Stock Returns. *Journal of Political Economy*, **111**(3), 642-685.
- Rachev, S.T., Mittnik, S., Fabozzi, F.F., Focardi, S.M. and Jasic, T.** (2007) *Financial Econometrics From Basics to Advanced Modeling Techniques*. New Jersey: John Wiley & Sons.
- Reilly, F.K. and Brown, K.C.** (2003) *Investment Analysis & Portfolio Management 7th Ed.* Mason, OH: Thompson Southwestern.
- Rosly, S.A. and Zaini, M.A.** (2008) Risk-return analysis of Islamic banks' investment deposits and shareholders' fund. *Managerial Finance*, **34**(10), 695-707.
- Selim, T. H.** (2008) An Islamic capital asset pricing model. *Humanomics*, **24**(2), 122-129.
- Shaikh, S.A.** (2009) Corporate Finance in an Interest Free Economy: An Alternate Approach to Practiced Islamic Corporate Finance. *Journal of Islamic Banking & Finance, International Association of Islamic Banks*.
- Sharpe, W.F.** (1964) Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance*, **XIX**(3), 425-442.
- Tomkins, C. and Karim, R.** (1987) The Sharī'ah and its Implications for Islamic Financial Analysis: An Opportunity to Study Interactions among Society, Organisations and Accounting. *The American Journal of Islamic Social Sciences*, **4**(1), 101-115.
- Verbeek, M.** (2004) *A Guide to Modern Econometrics* (2nd ed.). Sussex, England: John Wiley & Sons.
- Wei, K.C.** (1988) An asset-pricing theory unifying the CAPM and APT. *Journal of Finance*, **XLIII**(4), 881-892.
- Zaher, T.S. and Hassan, M.K.** (2001) A Comparative Literature Survey of Islamic Finance and Banking. *Financial Markets, Institutions & Instruments*, **10**, 155-199.

Appendix

I. Derivation of CAPM

Sharpe (1964) combines a risky asset i and the efficient market portfolio M to construct portfolio, p ; i is a constituent M . Assuming w and $(1 - w)$ as investment weights in i and M , he uses Markowitz's portfolio variance formula to derive risk of the portfolio.

$$\sigma_p = \sqrt{w^2\sigma_i^2 + (1-w)^2\sigma_M^2 + 2w(1-w)\rho_{iM}\sigma_i\sigma_M} \quad (A1)$$

where, σ_p is the standard deviation of the portfolio p ; σ_i^2 and σ_M^2 are the variances of stock i and portfolio M , respectively, and ρ_{iM} is the correlation between returns on i and M .

The expected return on p is given by

$$E(r_p) = wE(r_i) + (1-w)E(r_M) \quad (A2)$$

Change in risk and return of p with change in weight of investment in i , is given by

$$\frac{\partial\sigma_p}{\partial w} = \frac{w(\sigma_i^2 + \sigma_M^2 - 2w\rho_{iM}\sigma_i\sigma_M) - (\sigma_M^2 - \rho_{iM}\sigma_i\sigma_M)}{\sigma_p} \quad (A3)$$

and

$$\frac{\partial E(r_M)}{\partial\sigma_M} = \frac{E(r_M) - E(r_i)}{\sigma_M - \rho_{iM}\sigma_i} \quad (A4)$$

At the tangency point, $w=0$ and $\sigma_p = \sigma_M$, (A3) reduces to

$$\frac{\partial\sigma_M}{\partial w} = -[\sigma_M - \rho_{iM}\sigma_i] \quad (A5)$$

and (A4) equals

$$\frac{\partial E(r_M)}{\partial w} = [E(r_i) - E(r_M)] \quad (A6)$$

Dividing (A5) by (A6) gives risk-return relationship for tangency portfolio M ,

$$\frac{\partial E(r_M)}{\partial\sigma_M} = \frac{E(r_M) - E(r_i)}{\sigma_M - \rho_{iM}\sigma_i} \quad (A1)$$

(A7) represents slope of the investment opportunity curve (IOC) at the tangency point. Also, at the tangency point, the slope of IOC equals that of the CML, which is

$$b = \frac{[E(r_M) - r_f]}{\sigma_M}; \quad \because \sigma_r = \sigma_M, \text{ and } r = r_M \quad (A2)$$

Equating (A7) and (A8)

$$\begin{aligned} \frac{E[(r_M) - r_f]}{\sigma_M} &= \frac{E(r_M) - E(r_i)}{\sigma_M - \rho_{iM}\sigma_i}, \text{ gives} \\ &\equiv \frac{\rho_{iM}\sigma_i}{\sigma_M} = \left[\frac{E(r_i) - r_f}{E(r_M) - r_f} \right] \end{aligned} \quad (A9)$$

Further, $\frac{\rho_{iM}\sigma_i}{\sigma_M} = \frac{\sigma_{iM}}{\sigma_M^2} = \beta_i$, the standardized covariance of returns on i and M with respect to the variance of the return on M . β_i reflects the systematic risk of asset i . Rearranging (A9) gives the capital assets pricing model,

$$E(r_i) = r_f + \beta_i[E(r_M) - r_f]$$

II. Derivation of Black's CAPM

Black (1972) establishes equivalence of CAPM with and without a risk-free asset by deriving equilibrium relationship for individual investors. He contends that investor k constructs an efficient portfolio from two basic portfolios in order to minimize the variance of his or her portfolio, such that

$$\text{Minimize } \sigma_k^2 = \sum_{i=1}^N \sum_{j=1}^N w_{ik} w_{jk} \sigma_{ij}; \quad (A10)$$

subject to:

$$\sum_{i=1}^N w_{ik} E(r_i) = E(r_k), \text{ and } \sum_{i=1}^N w_{ik} = 1$$

where σ_k^2 is the variance of the portfolio of investor k , ($k = 1, 2, \dots, K$) and w_{ik} represent the weights assigned to stock i ($i = 1, 2, \dots, N$) in the efficient portfolio held by the investor k .

Applying Lagrange multipliers λ_{1k} and λ_{2k} in (A10) gives

$$\begin{aligned} \text{Minimize } \sigma_k^2 &= \sum_{i=1}^N \sum_{j=1}^N w_{ik} w_{jk} \sigma_{ij} \\ &\quad - 2\lambda_{1k} \left[\sum_{i=1}^N w_{ik} E(r_i) - E(r_k) \right] \\ &\quad - 2\lambda_{2k} \left[\sum_{i=1}^N w_{ik} - 1 \right] \end{aligned}$$

Taking the derivative of the Lagrangean with respect to w_{ik} gives

$$\sum_{j=1}^N w_{jk} \sigma_{ij} - 2\lambda_{1k} E(r_i) - 2\lambda_{2k} = 0 \quad (A3)$$

Let d_{ij} be the inverse of σ_{ij} matrix, and multiplying both sides by d_{ij} transforms (A11) into

$$w_{jk} = \lambda_{1k} \sum_{j=1}^N d_{ij} E(r_i) + \lambda_{2k} \sum_{j=1}^N d_{ij} \quad (\text{A4})$$

In (A12), reference to the investor k attaches to the Lagrange multipliers only, which signifies that investor k can create portfolio from two basic portfolios. Black asserts that all efficient portfolios are linear combinations of the two basic portfolios. Normalizing (A12) gives

$$w_{ik} = y_{Uk} w_{iU} + y_{Vk} w_{iV} \quad (\text{A13})$$

where U and V are the two basic portfolios, and

$$\begin{aligned} y_{Uk} &= \lambda_{1k} \sum_{i=1}^N \sum_{j=1}^N d_{ij} E(r_i) \\ y_{Vk} &= \lambda_{2k} \sum_{i=1}^N \sum_{j=1}^N d_{ij}; \\ w_{iU} &= \frac{\sum_{i=1}^N d_{ij} E(r_i)}{\sum_{i=1}^N \sum_{j=1}^N d_{ij} E(r_i)} \\ w_{iV} &= \frac{\sum_{i=1}^N d_{ij}}{\sum_{i=1}^N \sum_{j=1}^N d_{ij}}; \\ \sum_{i=1}^N w_{iU} &= 1; \quad \sum_{i=1}^N w_{iV} = 1; \quad \text{and } y_{Uk} + y_{Vk} = 1 \end{aligned}$$

where y_{Uk} and y_{Vk} denote investor k 's proportion of investment in portfolios U and V , respectively; w_{iU} and w_{iV} denote weights of stock i ($i = 1, 2, \dots, N$) in portfolios U and V , respectively. It follows that w_{ik} in (2.22) also must sum to 1.

Suppose that ω_{Mk} is the fraction of total wealth held by investor k then adding it up over all investors provides weight for each asset in the market portfolio

$$\begin{aligned} w_{iM} &= \left(\sum_{k=1}^K \omega_{Mk} w_{Uk} \right) w_{iU} \\ &\quad + \left(\sum_{k=1}^K \omega_{Mk} w_{Vk} \right) w_{iV}; \quad i = 1, 2, \dots, N \end{aligned} \quad (\text{A5})$$

where w_{iM} is the weight of asset i in market portfolio M , ω_{Mk} , the proportion of wealth invested by each investor k in market portfolio, w_{Uk} and w_{Vk} are the weights assigned by investor k to portfolios U and V , and w_{iU} and w_{iV} are the weights of stock i in U and V . Since market portfolio is a weighted combination of the basic portfolios U and V , and beta of market portfolio, β_M , is 1, which when combined with a zero-beta portfolio in an efficient portfolio e , yields

$$r_e = \beta_e r_M + (1 - \beta_e) r_z$$

III. Lintner Case I of Investment Choices

Lintner (1965) case I of investment choices states that investors can choose between holding cash and investing in a Single common stock. Assuming ω_0 as the wealth held by an investor, of which w_i is invested in stock i and $(1 - w_i)$ is held in cash. The expected total dollar return, y , on the investment is given by

$$y \omega_0 = w_i r_i \omega_0$$

or

$$y = w_i r_i \quad (\text{A15})$$

r_i is the return in stock i . The standard deviation of the returns equals

$$\sigma_y = w_i \sigma_{r_i}$$

or

$$w_i = \sigma_y / \sigma_{r_i} \quad (\text{A16})$$

Substituting (A16) into (A15) gives

$$y = r_i \left(\frac{\sigma_y}{\sigma_{r_i}} \right)$$

An investor can change his overall return y and overall risk σ_y as he may wish along the market opportunity line, which is equivalent of CML in CAPM. Both expected return and risk increase as the proportion of investment in i increases. r_i / σ_{r_i} , the reciprocal of the coefficient of variance of returns is the market price of risk.

نموذج تسعير الأصول الرأسمالية وتسعير الأدوات المالية الإسلامية

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المستخلص: إن تحريم الربا (الفائدة)، وتحريم تداول الأدوات المالية للشركات المحرمة (غير المتوافقة مع الشريعة) يفرض إعادة النظر في الفضاء الاستثماري الذي يقوم عليه نموذج تقييم الأصول الرأسمالية (Capital Asset Pricing Model - CAPM)؛ حيث أن أصولاً خالية من المخاطر ومحفظة سوقية تشتمل على أدوات للشركات المحرمة لا علاقة له بعالم الاستثمار القائم على الأسس والمبادئ الإسلامية. اقترحت الدراسة نوعين من (CAPM) نموذج تقييم الأصول الرأسمالية المتوافق مع الشريعة الإسلامية، والذي يستخدم محفظة سوقية تشتمل على أدوات متوافقة مع الشريعة فقط ولا يشتمل على أصول خالية من المخاطر. اختبرت الدراسة النماذج المقترحة مع النموذج التقليدي مع تطبيق لذلك في بورصة ماليزيا مستخدمةً بيانات تمتد مُدد بعضها إلى ثلاث سنوات، والبعض الآخر عشر سنوات. وأظهرت نتائج اختبار النماذج للبيانات ذات السنوات الثلاث تشابهاً ملحوظاً بين النماذج المقترحة المتوافقة مع الشريعة. وإلى جانب ذلك، فإن نموذج تقييم الأصول الرأسمالية المتوافق مع الشريعة قادر على شرح العوائد على الأسهم المتوافقة مع الشريعة بدقة مماثلة للنموذج التقليدي في شرحه للعوائد على جميع الأسهم.

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