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# Modeling the Premium and Contract Properties of Family *Takāful* (Islamic Life Insurance)

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***Abstract.*** The premium is a deterministic function to compensate for losses due to random events, and a crucial element for the operation of the standard insurance company. This paper focuses on the practice of family *takāful* and considers properties of the contract in determining the premiums comprehensively and in a way that makes them Sharīʿah-compliant. We have developed a new mortality derivative formula and model of the premium for an equity-linked policy (*unit-linked product*). We have adapted the option pricing of the Black-Scholes model for periodical premiums, taking into account a minimum death benefit, the value of surrender option, and maturity guaranteed payoff. We have also added some assumptions for the underlying asset prices following negative discrete dividend extensions from the dynamic escrowed model. The purpose is to obtain an unbiased option price for the underlying asset. We found it to be a satisfying product with a fair periodical premium, which has great flexibility in its features and complies comprehensively with Sharīʿah law.

***Keywords*:** Premium of family *takāful*, Equity-linked policy and Escrowed dynamic discrete dividend model.

**JEL Classification:** G12, G17, G22.

**KAUJIE Classification:** H13, I44, K1.

## 1. Introduction

The insurance industry plays an important role in any economy, and can help economic institutions share potential risks that occur in business operations and financing activities. Insurance not only facilitates economic transactions through the transfer of risk and indemnity, but can also promote financial intermediation (Ward & Zurburegg, 2000). Explicitly, the insurance industry promotes financial stability, mobilizes savings, facilitates trade and commerce, enables risk to be managed more efficiently, encourages loss mitigation, fosters efficient capital allocation, and can be seen to function as a substitute for government security programs (Skipper & Barfield, 2001).

The practice of insurance under common law has become a crucial issue from an Islamic perspective, with Islamic scholars promoting and practicing a type of Islamic insurance known as *takāful*. The main goal of both *takāful* and conventional insurance is to create and uphold a sense of solidarity, mutual responsibility, and brotherhood among parties involved on the basis of mutual co-operation, in the event of one of the parties experiencing an unfortunate event that would lead to suffering (Qureshi, 2011). However, there is a great difference in the practice of conventional insurance in the eyes of Islamic teachings; there are elements which are prohibited under Islamic law (Sharīʿah) such as interest (*ribā*), uncertainty (*gharar*), and speculation (*maysir*) (Pillsbury, 1998). In addition, the operational system that is applied in the practice of conventional life insurance does not conform to the rules of Sharīʿah. This is because the elements of *gharar* and *maysir* exist as consequences of uncertainty, gambling, and speculation from investment activities and the insurance contract (*ʿaqd*).

The *ʿaqd* is an important issue in the practice of insurance in an Islamic context. The *ʿaqd* contained in the existing insurance agreement can impact on the existence of uncertainty (*gharar*) and speculation (*maysir*). Therefore, scholars and Islamic economic experts have found a solution to the issue, meaning that it can be avoided. The first issue is the uncertainty (*gharar*) element arising from the contract used in conventional insurance; it is similar to *ʿaqd tabādulī* (buying and selling agreement) in *fiqh muʿāmalah* (Rusly & Ahmad, 2003).

Further, a significant issue in *takāful* is the determination of a premium for family *takāful* (life insurance) relating to the technique used to calculate the premium, which comprehensively fulfills the Sharīʿah requirement. In accordance with the terms of the buying and selling agreement, it should be clear with regard to the payment of premium and how much money is to be received by the policyholder at the maturity of the insurance contract. Legal issues (Sharīʿah) arise here because we cannot determine precisely the amount of premium to be paid, even if certain other conditions relating to the seller, buyer, *ʿaqd*, and the sum insured, can be calculated. The amount of insurance premium to be paid by the policyholder is dependent on a risk assessment, which relates to the probability of death (mortality rate), expected return rate, expected costs, and expected amount of claims – including premature death or the policyholder still being alive at the maturity of the insurance contract. Here the element of *gharar*, or uncertainty, is present (Ali, Odierno, & Ismail, 2008; Abd Rahman, Borhan, Ibrahim, Seman, & Ali, 2008). Based on the previous discussion regarding the issue of lawful and unlawful relations with insurance practice, the present paper focuses on equity-linked policies (*Unit-linked life insurance products*).

The equity-linked policy is one of the most sophisticated life insurance products to emerge in recent years. This product has presented new problems for actuaries because some of the issues are unsolvable from traditional actuarial approaches. The problem concerns the premium of minimum death benefit and the maturity benefit guarantee, as well as determining appropriate reserves for the guarantee value. Although there is an extensive actuarial literature dealing with this subject, the consensus seems to be that no completely satisfactory solution has been found to determine life insurance premium appropriately and accurately. In an insurance company, the insurance premium plays an important role, wherein it is based on the concept of pooling or loss sharing.

In the present paper, we develop a new model of the premium and new derivative mortality for *takāful* life insurance to determine a fair periodical premium adopted from the Black-Scholes option pricing model and escrowed dynamic model, specifically for equity-linked policies. This considers the mortality risk and properties of *takāful* life insurance comprehensively. This model is intended to be fully Sharīʿah compliant. This paper contributes to improving and solving a problem, which has still been under discussion until recent years, filling several gaps from previous studies including the surrender option value guaranteed before maturity.

The rest of the paper is structured as follows: Section 2 discusses the literature review related to premiums. The discussion focuses on the modeling of premium. Section 3 is the research methodology. Section 4 discusses the findings featuring the implementation of the model through numerical simulation, and testing the expected results. It provides a detailed explanation of the model and a discussion of the simulation results. Section 5 concludes the study, summarizing the findings and implications of the model. Lastly, section 6 covers the research contribution and limitations.

## 2. Literature Review

A number of previous studies have been conducted relating to the modeling of insurance premium rates, specifically for equity-linked policy considering the guaranteed asset value. The first was by Black and Scholes (1973) and Merton (1973), who developed a theory of option pricing and applied it to managing risks. In this theory, a financial guarantee is a part benefit under an insurance policy, known as an embedded option. Further, Brennan and Schwartz, (1976; 1979) and Boyle and Schwartz (1977) adopted and derived the option-pricing model of Black-Scholes-Merton (1973) into a regular and single premium. This aims to recognize that the benefits payable under an equity-linked policy asset value are equivalent to the guaranteed amount plus the value of an immediately exercisable call option on the reference portfolio, with an exercise price equal to the guaranteed amount.

Later, the findings of Brennan and Schwartz (1976; 1979) and Boyle and Schwartz (1977) influenced Delbaen (1990), who examined the periodical premium using a martingale approach to consider contingent claims for the valuation of an equity-linked policy, which had been introduced by Harrison and Kreps (1979). Delbaen was working towards tackling a problem that could be solved numerically using Monte Carlo methods. In addition, Bacinello and Ortu (1993) analyzed the problem of pricing insurance contracts where the benefits linked to the realization of a portfolio of equities and a minimum amount was guaranteed. They extended the model developed by Brennan and Schwartz (1976; 1979) and Delbaen (1990) to calculate endogenous minimum guarantees and apply the same approach to other traditional types of contracts.

More recently, Bacinello and Persson (2002) have evaluated a design and pricing model for equity-linked life insurance contracts incorporating stochastic interest. They applied a simple model of a financial market, such as mutual funds and default-free bonds, and restricted the study to two sources of uncertainty: risk related to the interest rate, and risk related to the mutual fund. They use the model developed by Heath, Jarrow, and Morton (1992), improving on the general framework used in Vasicek (1977) and Cox, Ingersoll, and Ross (1985). They found that the model developed could be generalized to previous pricing models based on deterministic interest rates. The new product created was simple to price and could easily be hedged – either by long positions in the mutual fund or by European call options on the same fund.

Further, Costabile (2013) computed the analytical periodical premium formula for an equity-linked policy without considering mortality risk. His formula showed a missing point for the calculation forms, and one variable applied this incorrect assumption, making the formula restricted. Due to this limitation, we added a variable mortality rate and correct some missing points for calculating the periodical premium. This was done because in practice, a long-term contract would increase expenses for an insurance company, thereby affecting the rate of the insurance premium. Therefore, the use of the martingale measure in the model should be considered alongside the future cost of risks.

As stated earlier, pricing equity-linked policies has been a major challenge faced by traditional actuarial approaches. In view of this, actuaries are expected to develop a new sophisticated approach to determine the rate of insurance premium, which must be sufficient and adequate to cover the amount of claims such as premature death, the guaranteed value of the surrender option, and the guaranteed maturity benefit if the policyholder is still alive when the insurance contract matures.

Some researches, Delbaen (1990); Bacinello and Ortu (1993); Bacinello and Persson (2002) and Costabile (2013), have frequently overlooked the aforementioned issues in discussions on life insurance premiums. One persistent problem in life insurance relates to fair premiums; periodical, annual, and total insurance premiums should be paid by the policyholder to cover their risks in future. Besides that, it is important to determine the premium that can give mutual benefit for both parties, namely the policyholder and *takāful* operator.

## 3. Methodology

The basic model of insurance that has advanced in practice is the Brownian motion model. The model was developed to solve a problem faced by insurance companies and corporations concerning risks in financial markets. The model created by Merton (1971) and Samuelsson, (1969), as extensions to the single horizon market models of Markowitz (1959) and Sharpe (1964), were concerned with the concepts of financial assets and markets and use continuous-time models with stochastic processes. Insurance companies need credible approaches to measure and manage risk exposure in-line with market price fluctuations including the foreign exchange rate, commodity prices, and stock prices. This model of insurance takes into account the issue of the financial market. Insurance firms began to invest their assets in two forms: some in risk-free assets and others in riskier assets. The basic assumptions include no transaction costs, no taxes, and the ability of assets to be traded continuously over time. In the stochastic differential equation for the prices of risky investment through geometric Brownian motion, the formula can be written as follows:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t) \quad S(0) = S_0, \tag{1}$$

Where, $\mu$ denotes the drift, the volatility of the process is notated by σ, and $W$ is a continuous time stochastic process (*Wiener Process*).

From equation (1), the price of the risk-free asset can be determined, written as follows:

$$dB(t) = rB(t)dt, \tag{2}$$

Where the continuous risk-free rate compounded is denoted by $r$, where $r$ is less than $\mu$. Under this model, the assets have prices that evolve continuously over time and are driven by the Brownian motion process.

In this paper, we derive and substitute some elements that are not applicable or not allowed in the Islamic financial system. Thus, we redefine the application of interest rate denoted by "*r*" in the formula as the appropriate probability cost of sellers in the financial market (Vogel & Hayes, 1998). Then, the model of premium considers Sharīʿah compliance in the practice of *takāful* life insurance, correcting the default approach to apply the system of *tabarruʿ* and payoff dividend for the insured if they are still alive at the maturity of the contract. We also utilize a standard actuarial notation (Bowers, Gerber, Hickman, Jones, & Nesbitt, 1997), *T(x)* which denotes the random future lifetime of the insured, aged $x$, with the probability density function existing and assuming that *T(x)* is independent of all financial random variables. Considering the mortality risk regarding the survival probability at the level of age i.e. ${}_t p_x$ is the probability that the insured is still alive at time $t$. Then, for the probability that the insured is not alive at time $t$ is denoted as ${}_n q_x$ or can be calculated by, $1 - {}_t p_x$.

In this setting, it is assumed that the policyholder has agreed to pay continuous payments, denoted by $P$, and that the payment is received at the beginning of the year, for $n$ years. This shows the regular premium that should be paid by the policyholder. This equally-spaced length of interval time $\Delta t$ (or length of interval time) is the value at $T$ divided by $n$. Thus, the date of premium payment periodically is $t_i = i.\Delta t$ where $i = 0,\ldots,n-1$. Furthermore, the premium paid periodically by the participant would be invested into the reference fund. Thus, following this assumption, the payoff obtained by the policyholder at the maturity of the contract is following the maximum reference fund and guarantee amount, the formula for which is given below:

$$GR(T) = CD\frac{e^{g(T-\Delta t)} - e^{g(\Delta t)}}{e^{g(\Delta t)} - 1} \tag{3}$$

In simple form, the above formula can be rewritten as follows:

$$GR(T) = \sum_{i=0}^{n-1} CDe^{g(T-t_i)} \tag{4}$$

Where *GR(T)* is the payoff obtained by the policyholder at the maturity of the insurance contract with time horizon $T$; *CD* denotes the contribution given in the reference fund, and $g$ is the value guarantee force for the appropriate probability cost by the seller. If $g = 0$, then the minimum value guarantee would be paid to the policyholder at the maturity of an insurance contract, following the time of the contract multiplied by the contribution given. Equation (4) indicates that periodical payment of premium starting at $i = 0, 1, 2, \ldots, n-1$, would affect the amount of contributions given to increase every year, and the maximum value received is based on the time of the insurance contract. This is assumed by ignoring the application of present value for future benefit and loss. Further, the value of the reference fund is denoted by *RF(T)*. To calculate benefits payable to the policyholder if alive at time $t$, we use $t = m+1, \ldots, n$ which is a quantity dependent on the value invested in the portfolio. The benefits payable can be set equal to *C(T) = max (RF(T),GR(T)*. This shows that the terminal value of the policy payoff is decomposed as the value of the minimum guaranteed rate plus the benefits payable at maturity of the contract. This uses a European call option written on the reference fund with an exercise price equal to a minimum of the value of the guaranteed rate. The formulation can be written as *C(T) = max (RF(T)-GR(T),0)*.

Further, for another possible decomposition for benefits payable at the maturity of the insurance contract is the European put option, which can be set equal to *P(T) = max (GR(T)-FR(T),0)*. In this setting, to create a fair value of periodical premium for *takāful* life insurance at the inception of the equity-linked policy, the decomposition of the European call and put option is considered. Based on financial economics, the evaluation framework is assumed following a completely frictionless market; there is no arbitrage opportunity because the expected value of the change in the asset for a whole period is equal to zero. This shows the existence of a unique equivalent martingale measure, denoted by $E^Q$, and each contingent claim is evaluated using discounting at the appropriate probability return of the seller in the financial market. In the current paper, we assume that there is no application of a risk-free rate because it is not allowed and does not comply with the Sharīʿah. Therefore, we substitute the application of a risk-free rate, denoted by *rf* in our equation, and replace it with an appropriate probability return of the seller in the financial market by using a new notation, *Rp* and the expected operator under the equivalent martingale measure $E^Q$ of contingent claims at the maturity of the contract.

We adopt and extend the Geometric Brownian Motion formula and assume the evolution of the reference fund value and period of premium payment with upward jumps of magnitude in the given contribution. Using equation (1), the new equation can be set equal to:

$$dRF(t) = RpRF(t)dt + \sigma RF(t)dW(t) \quad (5)$$

Thus, following the assumption in this setting, the periodical premium paid by policyholders can be written as follows:

$$RF(t_i) = RF(t_i)dt + CD \quad t_i < t < t_{i+1}, i = 0, \ldots, n-1 \quad (6)$$

Where $RpRF(t)$ is the appropriate opportunity return of the seller in a reference fund at time $t$, the volatility of the reference fund at time $t$ is denoted by $\sigma RF(t)$, and where $W(t)$ is a standard Geometric Brownian motion under a risk-neutral probability measure (*Wiener process*) and the contribution given is denoted by *CD*.

Then, the fair value of the insurance scheme using the expectation operator under an equivalent martingale measure can be obtained as the present value of the guaranteed amount and the value of the call option, formulated as follows:

$$E_t^Q(GR(T) + C(T)) = E_t^Q(GR(T)) + E_t^Q(C(T)) \quad (7)$$

Where, $E_t^Q(.)$ represents the expectation operator value at the start of the insurance scheme against contingent claims under a risk-neutral probability measure. $E_t^Q(GR(T))$ is computed using the discounting at the appropriate opportunity return of the seller from the investment fund in a fixed guarantee *GR*(*T*), as below:

$$E_t^Q(GR(T) = e^{-Rp(T)}CD\left(\frac{e^{g(T-\Delta t)} - e^{g(\Delta t)}}{e^{g(\Delta t)} - 1}\right)^2 \quad (8)$$

The computation of the call option value at the inception of insurance coverage is more complicated and difficult due to the upward jumps in the reference fund value dynamics. Hence, the issue of insurance coverage must be dealt with as an important part of the insurance product. In this setting, we also introduce a new mortality derivative and create a new formula for calculating the rate of mortality to determine a *tabarru*ʿ premium, which is derived from the basic mortality formula advanced in the practice of insurance companies (Bowers et al. 1997; Rejda, 2008), changing some parts which are not allowed in view of Islamic law. For this, we use the basic formula to determine mortality risk i.e. life function through a mortality table. It is calculated using the probability of life and death. The probability value of people being alive or dead at *t* can be written as below:

$$ {}_{n}p_{x} = \frac{l_{x+1}}{l_{x}} \text{ and } {}_{n}q_{x} = \frac{l_{x} - l_{x+1}}{l_{x}} = 1 - {}_{n}p_{x} \tag{9} $$

From equation (9) and (10), we found some missing points in the practice of insurance generally. We replace the statement "*fair value*", which comes from the sum of insurance coverage offered and multiplying it with the annual premium. In this setting, we change a term familiar to insurance companies using moving windows. Equation (9) can be simplified and set up equal to:

$$ {}_{n}q_{x} = \frac{l_{x=0} - l_{x+1}}{l_{x=0}} = 1 - {}_{n}p_{x} \tag{10} $$

Then, from equation (10), we find a margin from the probability value of death at time *t*. Following the above issue, this Sharīʿah-prohibited type of speculation is adopted by *takāful* life insurance to determine the risk premium. Thus, we eliminate the issue relating to the probability of life using moving windows. From equation (10), we create a new equation to calculate mortality risk. It can be seen as follows:

$$ {}_{n}\overline{q}_{x} = \frac{l_{x+t} - l_{x+1}}{l_{x+t}} = 1 - {}_{n}p_{x} \text{ where, } t = 0, 1, 2, \ldots, n. \tag{11} $$

Further, the total probability of an individual's death can be set equal to:

$$ {}_{n}\overline{q}_{x} = \sum_{i=0}^{n} \left( \frac{l_{x+t} - l_{x+1}}{l_{x+t}} \right) \tag{12} $$

*Proposition (1):* equations (10 and 11) indicate that the moving window on a new formula has a positive effect towards the probability value of death at the beginning of the year. This means that the application of the equation would reduce the risk of an insurance company in the operational and default probability of risks.

Further, we can use equation (12) risk premium (*tabarruʿ* premium) model and develop this to take account of sharing risk among participants:

$$ {}_{n}\widetilde{q}_{x} = \sum_{i=0}^{n} \left[ CD \frac{e^{g(T-\Delta t)} - e^{g(\Delta t)}}{e^{g(\Delta t)} - 1} \right] * 1 \left( \frac{1}{1+r} \right)^{n} * \left( {}_{n}\overline{q}_{x} \right) * n \tag{13} $$

Furthermore, to set up the *tabarruʿ* premium that should be paid by the participant, we get:

$$ {}_{n}\widetilde{q}_{x} = \max \left( \sum_{i=0}^{n} \left[ CD \frac{e^{g(T-\Delta t)} - e^{g(\Delta t)}}{e^{g(\Delta t)} - 1} \right] * 1 \left( \frac{1}{1+r} \right)^{n} * \left( {}_{n}\overline{q}_{x} \right) * n \right) \tag{14} $$

Where the *tabarruʿ* premium is denoted by ${}_{n}\widetilde{q}_{x}$, *CD* is the contribution given, *r* is appropriate probability cost by the seller, *n* is a time of the contract, *g* is the guaranteed value rate of return from the investment, ${}_{n}\overline{q}_{x}$ is the new mortality derivative applied moving windows.

*Proposition (2):* equations (13 and 14) show the contribution given, mortality derivative, appropriate probability cost by the seller and time of contract which offers a positive effect to the worth of the *tabarruʿ* premium. Nevertheless, at the same time it would be increase the sum of insurance coverage.

Following equation (14), the *tabarruʿ* premium would be paid by the participant at the beginning of the year using the maximum value of risk. In addition, the product created should consider the terminal wealth of participants through the sum of insurance coverage. Thus, if the policyholder needs the sum of the insurance to be large, then the participant should pay a higher *tabarruʿ* premium. It can be concluded that the sum of insurance coverage affects the rate of the *tabarruʿ* premium, and should be paid by the participant periodically. It is can be set up as follows:

$$ {}_n\widetilde{q}_x = \max\left( \sum_{i=0}^{n} \left[ CD \frac{e^{g(T-\Delta t)} - e^{g(\Delta t)}}{e^{g(\Delta t)} - 1} \right] * 1 \left( \frac{1}{1+r} \right)^n * \left( {}_n\overline{q}_x \right) * (n*k) \right) \tag{15} $$

where $k$ is the number of multiplication periods in the insurance contract.

Further, to determine the fair periodical premium, we compute an analytical approximation of the European call option on the underlying asset that pays fixed (*negative*) dividends at each premium payment period. The simple model for evaluating an option written on stock-paying fixed discrete dividends can be solved using the escrowed model. The escrowed model is based on the idea where future dividends are considered in advance, and the present value may be added to the current underlying asset price. Then, using adjusted current price, the dynamics of underlying asset prices without the upward jumps are induced at each premium payment date for the product. The formula can be set equal to:

$$ d\overline{RF}(t) = Rp\overline{RF}(t)dt + \sigma\overline{RF}(t)dW(t) \tag{16} $$

Where, $Rp\overline{RF}(t)$ is the appropriate opportunity return of the seller in a reference fund ignoring the upward jumps at time $t$, then, the volatility in a reference fund for the rate of return and ignoring the upward jumps at the time $t$ is denoted by $\sigma\overline{RF}(t)$, and $W(t)$ is a standard Geometric Brownian motion under a risk-neutral probability measure (*Wiener process*). Using equation (16) in the simple form, we get:

$$ \overline{RF}(0) = RF(0) + \sum_{i=1}^{n-1} CDe^{-Rp(t_i)} \tag{17} $$

Further, from equation (18), we get another value of the reference fund, written below:

$$ \overline{RF}(0) = CD\frac{1-e^{-Rp(T)}}{1-e^{-Rp(\Delta t)}} \tag{18} $$

*Proposition (3):* equation (17) shows the fluctuation of the reference fund with upward jumps of time equal to zero (0). This would increase the value of the reference fund at each time. If the time of the reference fund is equal to zero, then the value of reference fund is equal to *CD*. At the same time, equation (18) indicates the fluctuation of the reference fund without upward jumps at the beginning of the year equal to *CD*. Nevertheless, the total value of reference fund (equation 19) would be smaller than the reference fund with upward jumps at the end of the year.

By using adjusted current underlying asset prices, the European call option of the Black-Scholes can be written as follows:

$$ E_t^Q(C(T)) = \overline{RF}(0)\Phi(d_1(t)) - GR(T)e^{-Rp(T)}\Phi(d_2(t)) \tag{19} $$

In equation (19), the cumulative distribution function exists and the problem in the replicating portfolio at time $t$. The function of cumulative distribution can be seen as below:

$$ d\pm(t) = \frac{1}{\sigma\sqrt{T}}\left[ \log\left( \frac{\overline{RF}(0)}{GR(T)} \right) + \left( Rp \pm \frac{\sigma^2}{2} \right) T \right] \tag{19.1} $$

From equation (19 and 19.1), we get the cumulative distribution function of the standard normal random variable denoted as $\Phi(d\pm(t)$.

In addition, the understanding of the escrowed model in general is consistently biased in option prices (Bos, Gairat, & Shepeleva, 2003; Costabile, 2013). The reason is that the volatility of the adjusted underlying asset price process $\hat{\sigma}\overline{RF}(t)$ is different from the volatility of $\sigma RF(t)$. It is a true process and is too big for periods before the dividends are paid, due to the increased value of underlying asset prices. Following the theoretical approach to modifying the escrowed dynamic model, it can be written as follows:

$$\hat{\sigma}^2 \approx \sigma^2 - \sigma\sqrt{\frac{\pi}{2T}}\left\{\begin{array}{l} \dfrac{4e^{\frac{\delta^2}{2}}}{\overline{RF}(0)} CD \ \mathrm{X} \ \sum_{i=1}^{n-1} e^{-Rp(t_i)} \\ \left[\Phi(\delta) - \Phi\left(\delta - \sigma\dfrac{t_i}{\sqrt{T}}\right)\right] \\ -\dfrac{e^{\frac{Q^2}{2}}}{RF(0)} CD^2 \sum_{i,j=1}^{n-1} e^{-Rp(t_i - t_j)} \\ \left[\Phi(Q) - \Phi\left(Q - 2\sigma\dfrac{\min(t_i, t_j)}{\sqrt{T}}\right)\right] \end{array}\right\} \tag{20}$$

Where,

$$\delta = \Phi(d + (t)) = \frac{1}{\sigma\sqrt{T}}\left[\log\left(\frac{\overline{RF}(0)}{GR(T)}\right) + \left(Rp + \frac{\sigma^2}{2}\right)T\right] \tag{20.1}$$

And,

$$Q = \Phi(d - (t)) = \frac{1}{\sigma\sqrt{T}}\left[\log\left(\frac{\overline{RF}(0)}{GR(T)}\right) + \left(Rp - \frac{\sigma^2}{2}\right)T\right] \tag{20.2}$$

Following equation (20), the European call option embedded in the policy value at the maturity of the insurance contract can be evaluated through the Black-Scholes model, set equal to:

$$\hat{E}_t^Q(C(T)) \approx C_{BS}(\overline{RF}(0), GR(T), Rp, \hat{\sigma}, T) \tag{21}$$

Using equation (21) we insert the *muḍārabah* concept in the premium calculation of *takāful* life insurance, which can be written as follows:

$$CM = \frac{CDe^{-Rp(T)}\left(\dfrac{e^{g(T-\Delta t)} - e^{g(\Delta t)}}{e^{g(\Delta t)} - 1}\right)^2 + \hat{E}_t^Q(C(T))}{1 - e^{-Rp(T)} / 1 - e^{-Rp(\Delta t)}} \tag{22}$$

Where, *CM* represents the contribution *muḍārabah*, *CD* is the contribution given that would be paid by the policyholder at the beginning of the year and annually, *Rp* is the appropriate probability of return for the seller, *g* is the value of the guaranteed rate, and $\hat{E}_t^Q(C(T))$ is the value of the call option using Black-Scholes formula, considering the negative discrete dividend (see equation 21).

*Proposition (4):* equation (22) shows that the contribution *muḍārabah* is affected from the change of *CD* or the increasing contributions given (*CD*). Further, the changes of the appropriate probability return of seller (*Rp*) given positive effect on the value of *CM* and Meanwhile, the value of the guaranteed rate (*g*) given negative effect on the value of *CM.*

Further, the expenses of *takāful* life insurance can be formulated equally as:

$$C_{EX}=\left(\frac{CDe^{-Rp(T)}\left(\frac{e^{g(T-\Delta t)}-e^{g(\Delta t)}}{e^{g(\Delta t)}-1}\right)^{2}+\hat{E}_{t}^{Q}(C(T))}{1-e^{-Rp(T)}/1-e^{-Rp(\Delta t)}}\right)-\ E_{t}^{Q}(C(T)) \tag{23}$$

Where $C_{EX}$ denotes the administration cost spent by the insurance company, such as fees for agents and others, $CD$ is the contribution paid by the policyholder at the beginning of the year and annually, $Rp$ is the appropriate probability return of the seller, $g$ is the value of the guaranteed rate, $\hat{E}_t^Q(C(T))$ is the value of the call option using the Black-Scholes formula considering the discrete dividend to the increased value of the underlying asset price (see equation 21), $E_t^Q(C(T))$ is the value of the call option using the Black- Scholes formula without considering the calculation of discrete dividend (see equation 11).

*Preposition (5):* equation (23) shows the cost that should be spent by the policyholder for the life insurance product. The change of contribution given ($CD$) would be seen as a positive effect toward the value of expenses spent by the policyholder. Further, the effect of an appropriate probability return of the seller is positive; or, $Rp$ increases alongside $C_{EX}$. The increase of the guaranteed rate ($g$) value would decrease the value of $C_{EX}$.

Following the European call option embedded in the policy value at the maturity of the insurance contract through the Black-Scholes model in equation (21), the periodical premium can be set equal to:

$$P=\frac{CDe^{-Rp(T)}\left(\frac{e^{g(T-\Delta t)}-e^{g(\Delta t)}}{e^{g(\Delta t)}-1}\right)^{2}+({}_{n}\tilde{q}_{x})+C_{BS}(\overline{RF}(0),GR(T),Rp,\hat{\sigma},T)}{1-e^{-Rp(T)}/1-e^{-Rp(\Delta t)}} \tag{24}$$

Where $P$ represents the periodical premium, the contribution given is denoted by $CD$, $g$ is the guaranteed value, $Rp$ is the appropriate probability return of the seller, ${}_{n}\tilde{q}_{x}$ is the *tabarru*ʿ premium using the derivative mortality risk developed. After that, the European call option is embedded under the equity-linked policy until the maturity of the insurance contract and this can be evaluated by the Black-Scholes model and dynamic Escrowed model (see equation 22). Then, the annual level premium payment paid by the policyholder at the beginning of the year can be written as follows:

$$ALP=Max\left(\frac{CDe^{-Rp(T)}\left(\frac{e^{g(T-\Delta t)}-e^{g(\Delta t)}}{e^{g(\Delta t)}-1}\right)^{2}+({}_{n}\tilde{q}_{x})+\hat{E}_{t}^{Q}(C(T))}{1-e^{-Rp(T)}/1-e^{-Rp(\Delta t)}}\right) \tag{25}$$

Where $ALP$ represents the annual level premium of the *takāful* life insurance paid by the policyholder, $CD$ is the contribution given, annually paid by the policyholder, $Rp$ is the appropriate probability return of the seller, $T$ is a time of the contract's maturation, and $g$ is the value of the guarantee. $\hat{E}_t^Q(C(T))$ is the European call option embedded in the policy value at the maturity of the insurance contract and ${}_{n}\tilde{q}_{x}$ is the *tabarru*ʿ premium using the derivative mortality risk developed.

Equation (25) indicates that the annual premium paid is a maximum value from *muḍārabah* plus the *tabarru*ʿ contribution. The annual value of the *muḍārabah* contribution includes insurance company expenses. It can then be concluded that the formula developed in this study includes the properties of the *takāful* business model, which consists of *muḍārabah* and *tabarru*ʿ contracts.

*Proposition (6):* equation (25) indicates the periodical premium paid by the participant for the equity - linked policy. The contribution given (*CD*) is arrived at from the appropriate probability of return for the seller (*Rp*), European call option embedded in the policy value at the maturity of the insurance contract $\hat{E}_t^Q(C(T))$ and the *tabarru*ʿ premium ${}_n\tilde{q}_x$ - these have a positive effect on the periodical premium (*P*). In other words, if the value of *CD* and *Rp* increase then the periodical premium also increases. Further, the value of the guaranteed rate is negatively affected by the periodical premium; this means that if the guaranteed rate increases, then the periodical premium will decrease.

## 4. Findings Discussion

In the present paper, we apply a numerical simulation for the analysis and prove the equation developed. The numerical simulation was performed using MATLAB R2015. In this setting, we assume (1) the investment component named 'contribution given' from the periodical premium paid by the policyholder at the inception of the equity-linked policy is $100, (2) the insurance premium would be paid annually at the beginning of the year, (3) premature death would happen at the end of the year, (4) the mortality rate of Indonesia (2011) would be used and selected for males aged 30 to 35, and (5) the guaranteed value from 'contribution deemed' is assumed negative returns considering the discrete dividends. Then, the numerical analysis would follow the parameters listed below:

The periodical insurance payment - *n* 5

- The value of contribution given – *CD* $100
- The appropriate probability returns of the seller – *Rp* 0.035
- The finite time horizon - *T* 5
- The guaranteed rate - *g* 0.000
- The volatility of reference fund - *σ* 0.100

Further, we also examine the value of the guaranteed rate *g*, volatility of the reference fund *σ*, the appropriate probability cost of the seller *r*, and the appropriate probability return of the seller *Rp*, calculated following a fluctuating value of gross domestic product. It is intended to comply fully with the Sharīʿah.

Using equations 3 and 4, with the additional assumption that various values of the guaranteed rate $g = 0$ until $g = 0.007$, and the time horizon is 5 years, then the outcome can be seen as follows:

**Table (1) The payoff that would be received by the policyholder at the maturity of the contract with fluctuation of the value guaranteed**

| Rate Guaranteed | Time Horizon | | | | | Payoff at the maturity |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | |
| $g = 0.000$ | 100.00 | 100.00 | 100.00 | 100.00 | 100.00 | 500.00 |
| $g = 0.001$ | 100.50 | 100.40 | 100.30 | 100.20 | 100.10 | 501.50 |
| $g = 0.002$ | 101.01 | 100.80 | 100.60 | 100.40 | 100.20 | 503.01 |
| $g = 0.003$ | 101.51 | 101.21 | 100.90 | 100.60 | 100.30 | 504.52 |
| $g = 0.004$ | 102.02 | 101.61 | 101.21 | 100.80 | 100.40 | 506.04 |
| $g = 0.005$ | 102.53 | 102.02 | 101.51 | 101.01 | 100.50 | 507.57 |
| $g = 0.006$ | 103.05 | 102.43 | 101.82 | 101.21 | 100.60 | 509.10 |
| $g = 0.007$ | 103.56 | 102.84 | 102.12 | 101.41 | 100.70 | 510.64 |

Table 1 indicates the fluctuation of guaranteed value increasing the payment received by the policyholder at the maturity of the contract. Where the rate of guarantee is $g = 0.000$, with time horizon *T*, it can be seen that in the first year the value of the contribution paid by the policyholder is fixed until five years at $100, and the payoff at the maturity is $500. Further, when $g = 0.001$, with time horizon *T*, the value of contribution given paid by policyholder fluctuates until five years starting at $100.50 at the beginning of the first year, while the payoff at maturity increases to \$501.50. Next, for the guaranteed rate $g = 0.003$, the value of contribution paid by the policyholder fluctuates until five years starting at \$101.01 at the beginning of the first year while the payoff at maturity increases to \$503.01. It can be concluded that the rate guarantee ($g$) has a positive effect on the value of the payoff at the maturity of the contract. In other words, when the guarantee value increases, the value paid at the maturity of the contract is higher. This is different from the payment of contribution given by the policyholder. It can be seen as below:

**Table (2) The value of contributions paid by the policyholder including the discounting factor**

| Rate Guaranteed | Time Horizon | | | | | Value of Contribution Deemed Paid |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | |
| $g = 0.000$ | 100.00 | 96.56 | 93.24 | 90.03 | 86.94 | 466.77 |
| $g = 0.001$ | 100.00 | 96.56 | 93.24 | 90.03 | 86.94 | 466.77 |
| $g = 0.002$ | 100.00 | 96.56 | 93.24 | 90.03 | 86.94 | 466.77 |
| $g = 0.003$ | 100.00 | 96.56 | 93.24 | 90.03 | 86.94 | 466.77 |
| $g = 0.004$ | 100.00 | 96.56 | 93.24 | 90.03 | 86.94 | 466.77 |
| $g = 0.005$ | 100.00 | 96.56 | 93.24 | 90.03 | 86.94 | 466.77 |
| $g = 0.006$ | 100.00 | 96.56 | 93.24 | 90.03 | 86.94 | 466.77 |
| $g = 0.007$ | 100.00 | 96.56 | 93.24 | 90.03 | 86.94 | 466.77 |

Table 2 describes the value of contributions paid by the policyholder including the discounting factor with the guaranteed value $g = 0$ until $g = 0.007$ and time horizon $T = 5$ years. The value of the contributions given at the beginning of the year is \$100, decreasing for the next year until the contract matures, i.e. \$86.94. Nevertheless, the total value of contributions paid by the policyholder over 5 years is \$466.77. It indicates that fluctuations of the guaran-teed value do not affect the value of contribution given at the beginning of the year, i.e. \$100. However, the discounting factor affects the decrea-sing value of contributions to be paid annually following the change of time horizon for 5 years.

Further, using equation (8) the inception of the insurance scheme against contingent claims under the assumption of risk-neutrality (using a discounting factor at the appropriate opportunity return of the seller from the investment fund in a fixed guarantee) is obtained as shown in table 3.

Table 3 illustrates the fixed value guarantee at the inception of the insurance scheme against contingent claims using the discounting factor at the appropriate opportunity return of the seller. We use three panels to test a fluctuation of the guaranteed value towards the guaranteed payoff at the maturity of the insurance contract i.e. $g = 0.000$, $g = 0.003$ and $g = 0.007$, as well as three parameters for fluctuating values of appropriate probability returns of the seller denoted by *Rp,* i.e. $Rp = 0.035$ until $Rp = 0.045$, with the time horizon denoted by $T = 5$ years. The results show that, with the value of guarantee $g = 0.000$ and $Rp = 0.035$, the guaranteed payoff received by the policyholder is \$419.73. Further, while $g = 0.000$ and the value of $Rp$ is changed to 0.040, the guaranteed payoff received by the policyholder decreases from \$419.73 to \$409.37. This indicates that when the guaranteed value is equal to zero, then the fluctuation of the appropriate opportunity return of the seller negatively affects the guaranteed payoff received by the policyholder at the maturity of the insurance contract following time horizon $T$.

**Table (3) The value of the fixed guaranteed at the inception of the insurance scheme against contingent claims under risk-neutral probability**

| The Guaranteed Value | The Fluctuation Value of Appropriate Probability Return of the Seller | Guaranteed Payoff |
|---|---|---|
| $g = 0.000$ | $Rp = 0.35$ | 419.73 |
| | $Rp = 0.40$ | 409.37 |
| | $Rp = 0.45$ | 399.26 |
| | $Rp = 0.50$ | 389.40 |
| | $Rp = 0.55$ | 379.79 |
| | $Rp = 0.60$ | 370.41 |
| $g = 0.003$ | $Rp = 0.35$ | 431.22 |
| | $Rp = 0.40$ | 420.57 |
| | $Rp = 0.45$ | 410.19 |
| | $Rp = 0.50$ | 400.06 |
| | $Rp = 0.55$ | 390.18 |
| | $Rp = 0.60$ | 380.55 |
| $g = 0.007$ | $Rp = 0.35$ | 447.04 |
| | $Rp = 0.40$ | 436.01 |
| | $Rp = 0.45$ | 425.24 |
| | $Rp = 0.50$ | 414.74 |
| | $Rp = 0.55$ | 404.50 |
| | $Rp = 0.60$ | 394.51 |

Using the second panel of the guaranteed value $g = 0.003$ and the fluctuation value of appropriate probability returns of the seller similar to that in the previous discussion, when, $Rp = 0.035$, the guaranteed payoff received by the policyholder at the maturity of an insurance contract is \$431.22. While when $g = 0.003$ and the value of $Rp$ is changed to 0.040, the guaranteed payoff received by the policyholder decreases from \$431.22 to \$420.57. Then, in the third panel the guaranteed value is $g = 0.007$, and the fluctuation value of appropriate probability of return to the seller $Rp = 0.035$, the guaranteed payoff received by the policyholder at the maturity of the contract is \$447.04. While when $g = 0.007$, the value of $Rp$ is changed to 0.040, the guaranteed payoff to the policyholder decreases from \$447.04 to \$436.01. Based on this result, it can be concluded that the fluctuation value of appropriate probability return of the seller has a negative effect on the guaranteed payoff at the maturity of the contract. In other words, any increase in the value of appropriate probability return of the seller would decrease the guaranteed payoff at the maturity of the insurance contract. Afterward, the change of the guarantee value gives a positive effect to the guaranteed payoff. It can be seen that increasing the guarantee value from $g = 0.000$ to $g = 0.003$, and $g = 0.007$, positively affects the guaranteed payoff received by the policyholder at the maturity of the contract.

Following equations (13) and (14), we can calculate the effect of a fluctuating variable toward the change of *tabarru*ʿ premium for periodical payments following the time horizon. Then, it is possible to determine the annual *tabarru*ʿ premium and total *tabarru*ʿ premium paid for time horizon $T = 5$ years. The results are shown below:

**Table (4) The annual *tabarru*ʿ premium, total *tabarru*ʿ premium and the effect of the change of the time horizon $T$ = 5 years**

| The change of variable | Time Horizon | | | | | Annual *Tabarru*ʿ Premium | Total *Tabarru*ʿ Premium |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | | |
| $r$ = 0.035, $g$ = 0.000 | 14.39 | 13.90 | 13.43 | 12.98 | 12.54 | 14.39 | 67.24 |
| $r$ = 0.035, $g$ = 0.003 | 14.52 | 14.03 | 13.55 | 13.10 | 12.65 | 14.52 | 67.85 |
| $r$ = 0.035, $g$ = 0.007 | 14.70 | 14.20 | 13.72 | 13.25 | 12.81 | 14.70 | 68.67 |
| $r$ = 0.050, $g$ = 0.000 | 14.18 | 13.51 | 12.86 | 12.25 | 11.67 | 14.18 | 64.48 |
| $r$ = 0.050, $g$ = 0.003 | 14.31 | 13.63 | 12.98 | 12.36 | 11.77 | 14.31 | 65.06 |
| $r$ = 0.050, $g$ = 0.007 | 14.49 | 13.80 | 13.14 | 12.51 | 11.92 | 14.49 | 65.85 |
| $r$ = 0.060, $g$ = 0.000 | 14.05 | 13.25 | 12.50 | 11.80 | 11.13 | 14.05 | 62.73 |
| $r$ = 0.060, $g$ = 0.003 | 14.18 | 13.37 | 12.62 | 11.90 | 11.23 | 14.18 | 63.30 |
| $r$ = 0.060, $g$ = 0.007 | 14.35 | 13.54 | 12.77 | 12.05 | 11.37 | 14.35 | 64.07 |

**Note:** The sum of insurance coverage would be $2500.00

Based on Table 4 the *tabarru*ʿ premium is determined by using two panels to investigate the effect of fluctuating variables in the model. In the first panel we use the appropriate probability cost of the seller denoted by $r$ = 0.035, and for the guaranteed rate we use $g$ = 0.000, $g$ = 0.003, and $g$ = 0.007. The result shows the periodical premium with $r$ = 0.035 and $g$ = 0.000 – the *tabarru*ʿ premium at the beginning of the year is $14.39 and at the second year it decreases from $14.39 to $13.90, until the fifth year it becomes $12.54. The annual *tabarru*ʿ premium obtained is $14.39, and the total *tabarru*ʿ premium paid by the policyholder at the time horizon $T$ = 5 years is $67.24 with the sum of insurance coverage being $2500.00. Besides that, for $g$ = 0.003 and $r$ = 0.035, the *tabarru*ʿ premium at the beginning of the year is $14.52, then, in the second year it decreases to $14.03, until the fifth year it becomes $12.65. Further, the annual *tabarru*ʿ premium is $14.52, and the total *tabarru*ʿ premium that would be paid by the policyholder at the time horizon $T$ = 5 years is $67.85. Meanwhile, for $g$ = 0.007 and $r$ = 0.035, the *tabarru*ʿ premium at the beginning of the year is $14.70, then, at the second year it falls to $14.20, until the fifth year it becomes $12.81. Furthermore, the annual *tabarru*ʿ premium is $14.70, and the total *tabarru*ʿ premium that would be paid by the policyholders at time horizon $T$ = 5 years is $68.67. Following the outcome from the first panel in the setting, we can see that the effect of a change in the variable guarantee $g$ is that it causes an increase in the periodical *tabarru*ʿ premium for each time horizon, the annual *tabarru*ʿ premium, and the total *tabarru*ʿ premium paid by the policyholder until the maturity of the insurance contract.

Further, in the second panel, we change the appropriate probability cost of the seller to $r$ = 0.050, while three levels of the guaranteed rate were used: $g$ = 0. 000, $g$ = 0. 003, and $g$ = 0. 007. The outcome shows the periodical premium. For $r$ = 0.050 and $g$ = 0.000, the *tabarru*ʿ premium at the beginning of the year is $14.18 and at the second year it decreases to $13.51 until the fifth year, it becomes $11.67. The annual *tabarru*ʿ premium obtained is $14.18, and the total *tabarru*ʿ premium paid by the policyholder at time horizon $T = 5$ years is $64.48 with the sum of insurance coverage being $2500.00. Besides that, for $g = 0.003$ and $r = 0.050$, the *tabarru*ʿ premium at the beginning of the year is $14.31 and at the second year it decreases f to $13.63 until the fifth year it becomes $11.77. Further, the annual *tabarru*ʿ premium is $14.31, and the total *tabarru*ʿ premium paid by the policyholder at time horizon $T = 5$ years is $65.06. Meanwhile, for $g = 0.007$ and $r = 0.050$, the *tabarru*ʿ premium at the beginning of the year is $14.49 and at the second year it decreases $13.80 until the fifth year it becomes $11.92. The annual *tabarru*ʿ premium here is $14.49, and the total *tabarru*ʿ premium paid by the policyholder at the time horizon of 5 years is $65.85.

Based on the result using the two-panel parameters to investigate the effect of changing *tabarru*ʿ premium that consist of annual *tabarru*ʿ premium, total *tabarru*ʿ premium and fluctuation of the *tabarru*ʿ premium for each time horizon, we obtain some interesting findings. By changing the value of the appropriate probability cost of the seller, $r = 0.035$, $r = 0.050$, and $r = 0.060$ with the guaranteed rate $g = 0.000$, the results indicate that the *tabarru*ʿ premium paid periodically by the policyholder falls over time (for $r = 0.035$, it is $14.39, for $r = 0.050$, it is $14.18), and at the same time negatively affects the total *tabarru*ʿ premium paid over 5 years. It can be concluded that the appropriate probability cost by the seller has a negative effect on the periodical premium and total premium paid. In other words, if the appropriate probability of the seller increases, then the periodical *tabarru*ʿ premium and total *tabarru*ʿ premium decreases. Hence, we also find in this setting that the effect of the application appropriates the probability cost of seller similar to the practice of interest in conventional insurance. This indicates that we have created a *tabarru*ʿ premium that is fair and competitive, similar to the practice of conventional insurance in the calculation of risk premiums. The advantage of this product is the flexibility in choosing *tabarru*ʿ premium as a charity fund or compensation for sharing risk among policyholders.

Further, from equation (15) we introduce flexibility in our product. For instance, if the participant chooses to increase the sum of insurance coverage, then the option to increase the sum of insurance would affect the increasing *tabarru*ʿ premium and automatically give a high annual level premium. By multiplying the standard sum of insurance coverage denoted by $k$, the policyholder can make a choice based on the change of variable $k$. The output obtained is shown in Table 5.

Table 5 shows the numerical testing in this setting divided into two panels. In the first panel, $n = 5$, $r = 0.035$, $g = 0.000$, and variable $k = 1$, the annual *tabarru*ʿ premium is $14.39. For the total *tabarru*ʿ premium, called the non-discounting factor, it is $71.95 at 5 years, while the total *tabarru*ʿ premium on the discounting factor is $67.24, and the sum of insurance coverage is $2,500.00. Hence, we find that the excess of *tabarru*ʿ premium from discounting and non-discounting is as much as $4.71. Further, the change of variable $k = 1.5$ results in an increase in the *tabarru*ʿ premium paid by the policyholder from $14.39 to $21.58. The same is true for the total *tabarru*ʿ premium non-discounting as it increases from $71.95 to $107.92, while the total *tabarru*ʿ premium on the discounting factor also increases from $67.24 to $100.86. The excess *tabarru*ʿ premium becomes $7.06, with the sum of insurance coverage increasing from $2,500.00 to $3,750.00. This indicates that changes in variable $k$ would result in an increase in the annual *tabarru*ʿ, total *tabarru*ʿ non-discounting and discounting, excess of *tabarru*ʿ premium, and sum of insurance coverage.

**Table (5) The annual and total *tabarru*ʿ premium paid and changes to the sum of insurance coverage based on variation in variable k.**

| Panels | Change of Variable | Annual *Tabarru*ʿ | Total *Tabarru*ʿ (Non-Discounting Factor) | Total *Tabarru*ʿ (Discounting Factor) | Excess of the *Tabarru*ʿ Premium | Sum of the Insurance Coverage |
|---|---|---|---|---|---|---|
| Panel 1 | k = 1 | 14.39 | 71.95 | 67.24 | 4.71 | 2,500.00 |
| | k = 1.5 | 21.58 | 107.92 | 100.86 | 7.06 | 3,750.00 |
| | k = 2 | 28.78 | 143.89 | 134.48 | 9.41 | 5,000.00 |
| | k = 2.5 | 35.97 | 179.86 | 168.10 | 11.76 | 6,250.00 |
| | k = 3 | 43.17 | 215.84 | 201.72 | 14.11 | 7,500.00 |
| | k = 3.5 | 50.36 | 251.81 | 235.34 | 16.46 | 8,750.00 |
| | k = 4 | 57.56 | 287.78 | 268.97 | 18.82 | 10,000.00 |
| Panel 2 | k = 1 | 14.18 | 70.92 | 64.48 | 6.44 | 2,500.00 |
| | k = 1.5 | 21.28 | 106.38 | 96.72 | 9.66 | 3,750.00 |
| | k = 2 | 28.37 | 141.83 | 128.96 | 12.88 | 5,000.00 |
| | k = 2.5 | 35.46 | 177.29 | 161.19 | 16.10 | 6,250.00 |
| | k = 3 | 42.55 | 212.75 | 193.43 | 19.32 | 7,500.00 |
| | k = 3.5 | 49.64 | 248.21 | 225.67 | 22.54 | 8,750.00 |
| | k = 4 | 56.73 | 283.67 | 257.91 | 25.76 | 10,000.00 |
| Panel 3 | k = 1 | 13.92 | 69.59 | 61.06 | 8.53 | 2,500.00 |
| | k = 1.5 | 20.88 | 104.39 | 91.59 | 12.79 | 3,750.00 |
| | k = 2 | 27.84 | 139.18 | 122.13 | 17.06 | 5,000.00 |
| | k = 2.5 | 34.80 | 173.98 | 152.66 | 21.32 | 6,250.00 |
| | k = 3 | 41.76 | 208.78 | 183.19 | 25.59 | 7,500.00 |
| | k = 3.5 | 48.71 | 243.57 | 213.72 | 29.85 | 8,750.00 |
| | k = 4 | 55.67 | 278.37 | 244.25 | 34.12 | 10,000.00 |

Note: Panel 1 $n = 5$, $r = 0.035$ and $g = 0.000$

Note: Panel 2 $n = 5$, $r = 0.050$ and $g = 0.000$

Note: Panel 3 $n = 5$, $r = 0.070$ and $g = 0.000$

Further, for the second panel, where $n = 5$, $r = 0.050$, and $g = 0.000$, we change the value of $r$, defined as the appropriate probability cost of the seller, to 0.050, and the variable $k$ remains set to 1. The annual *tabarru*ʿ premium then, is \$14.18. The total *tabarru*ʿ premium non-discounting factor is \$70.92, and the total *tabarru*ʿ premium on the discounting factor is \$64.48, while the sum of insurance coverage is \$2,500.00. The excess of the *tabarru*ʿ premium from discounting and non-discounting is \$6.44. Further, the change of variable $k = 1.5$ results in an increase in the *tabarru*ʿ premium paid by the policyholder from \$14.18 to \$21.28. The total *tabarru*ʿ premium discounting and non-discounting factor, the excess *tabarru*ʿ premium and the sum of insurance coverage also increase. In the third panel, where $n = 5$, $r = 0.070$, $g = 0.000$, and the variable $k$ is still set equal to 1, the annual *tabarru*ʿ premium is \$13.92. The total *tabarru*ʿ premium non-discounting factor is \$69.592, and the total *tabarru*ʿ premium on the discounting factor is \$61.06, while the sum of insurance coverage is \$2,500.00. The excess of the *tabarru*ʿ premium from discounting and non-discounting is \$8.53.

From the two panels we illustrate the flexibility of our product and the effect of the change of annual *tabarru*ʿ, total *tabarru*ʿ premium, an excess of *tabarru*ʿ premium and sum of insurance coverage. We found that the change of variable $k$ positively affects the *tabarru*ʿ premium paid and the sum of insurance coverage in the policy insurance, having an overall positive effect on the *tabarru*ʿ premium paid by the policyholder in the insurance agreement. This can be compared with the effects observed when the value of the appropriate probability cost of the seller, $r$ is changed. In this case, a change in the value of $r$ negatively affects the *tabarru*ʿ premium. For instance, we can see that the annual *tabarru*ʿ premium decreases over time (see, when $r = 0.035$, it is \$14.39, and when $r = 0.050$, it is \$14.18). Nevertheless, we obtain an increased excess of *tabarru*ʿ premium (see, when $r = 0.035$, it is \$4.71, and when $r = 0.050$, it is \$6.44), indicating that a change in the value of $r$ results in an increase in the excess of *tabarru*ʿ premium at the finite time horizon or at the maturity of the contract.

Using equation (19), at inceptions we can determine the value of the insurance scheme for an equity-linked policy (*unit-linked product*) with a finite time horizon of 5 years. Then, the European call option of the Black-Scholes model, using the adjusted current underlying asset price without the assumption of a negative discrete dividend, can be used as shown In Table 6:

**Table (6) The value of the inception insurance scheme for equity-linked policy (unit-linked product) with the finite time horizon $T = 5$ years**

| Panels | Change of the Variable | Value of the Call Option | Value of the Guaranteed Rate | Value of the Insurance Contract |
|---|---|---|---|---|
| Panel 1 | $g = 0.000$ | 68.93 | 419.73 | 488.66 |
| $Rp = 0.035$ | $g = 0.003$ | 66.50 | 423.53 | 490.02 |
| $\sigma = 0.10$ | $g = 0.007$ | 63.30 | 428.66 | 491.96 |
| Panel 2 | $g = 0.000$ | 79.60 | 389.40 | 469.00 |
| $Rp = 0.050$ | $g = 0.003$ | 77.09 | 392.92 | 470.01 |
| $\sigma = 0.10$ | $g = 0.007$ | 73.77 | 397.68 | 471.45 |
| Panel 3 | $g = 0.000$ | 93.99 | 352.34 | 446.33 |
| $Rp = 0.070$ | $g = 0.003$ | 91.44 | 355.53 | 446.97 |
| $\sigma = 0.10$ | $g = 0.007$ | 88.05 | 359.84 | 447.89 |

Table 6 describes the value at inception of the equity-linked policy. By using the two panels and changing the variable guaranteed rate $g$, and appropriate probability of return of the seller, $Rp$, we find that in the first panel, with $Rp = 0.035$, and $g = 0.000$, the value of the call option is \$68.93, the value of the guaranteed rate paid to the policyholder at the maturity of the contract is \$419.73, and the value of the insurance contract is \$488.66. Then, if the guaranteed rate changes to $g = 0.003$, the value of the call option decreases to \$66.50, the value of the guaranteed rate received by the policyholder at the maturity of the contract increases to \$423.53, and the insurance contract value becomes \$490.02. Furthermore, for the value of the guaranteed rate $g = 0.007$, the value of the call option decreases from \$66.50 to \$63.30, the value of the guaranteed rate received by the policyholder at the maturity of the contract increases to \$428.66, and the insurance contract value becomes \$491.96. This indicates that the change of the guaranteed value results in a negative effect on the call option value, where the increasing rate of guarantee, decreases the value of the call option. Meanwhile, this is in contrast to its effect upon the value guaranteed payoff that would be paid to the policyholder at the maturity of the contract, where the change of guaranteed value has a positive effect upon it. In other words, if the guaranteed rate increases, the value guaranteed payoff received by the policyholder at the maturity of the contract would also increase.

In the second panel, it can be seen that when the appropriate probability of return of the seller $Rp$ = 0.050, and guaranteed rate $g$ = 0.000, the value of the call option is $79.60, and the value of guarantee paid to the policyholder at the maturity of the contract is $389.40, while the value of the insurance contract is $469.00. Then, if the guaranteed rate is changed to $g$ = 0.003, the value of call option decreases from $79.60 to $77.09, and the value of guaranteed payoff to the policyholder at the maturity with time horizon 5 years rises from $389.40 to $392.92, while the insurance contract value increases from $469.00 to $470.01. Further, for the value of the guaranteed rate $g$ = 0.007, the value of call option also decreases to $73.77, the value of the guaranteed payoff received by the policyholder at the maturity of the contract rises to $397.68, and the insurance contract value becomes $471.45.

From the two panels, we find that the appropriate probability of return of the seller has a positive effect on the value of the call option. This means that when the appropriate probability of return of the seller increases, the value of the call option at the inception of the equity-linked policy would also rise. Nevertheless, this is in contrast to its effect upon the value of guarantee received by the participant at the maturity of the contract and the contract value on the insurance scheme where it has a negative effect upon them. In other words, increasing the value of the appropriate probability return of the seller would cause a decrease in the value of guarantee received by the participant at the maturity of the contract and in the value of the contract.

In this setting, we also consider the concept of a *muḍārabah* contract in calculating an investment fund collected from the premium payment of the policyholder. Using equations (22) and (23), the value of the *muḍārabah* contribution, the expenses can be seen in Table 7.

Table 7 is divided into two panels with each panel being further divided into two sub sections. The aim is to facilitate comparisons by change of variables and parameters in the panels applied. In the first panel (panel 1A) we use the appropriate probability return of seller, $Rp$ = 0.035, the volatility of the reference fund is $\sigma = 0.15$, and the guaranteed rate is $g$ = 0.000. We obtain the value of the investment fund as $86.86, the expense value is $28.87 and the value of *muḍārabah* including expenses is $115.72. Then, if there is a change in the variable guarantee rate $g$ = 0.003, the value of the investment fund decreases to $84.74, the expense value becomes $22.83, and the value of *muḍārabah* including expenses is $107.57. When the guarantees rate changes to $g$ = 0.007, the value of the investment fund also decreases from $84.74 to $81.94, the expense value becomes $14.42, and the value of *muḍārabah* including expenses is $96.36. Hence, it has been demonstrated that the change of the guarantee rate variable has a negative effect on the value of the investment fund, expenses, and *muḍārabah* contribution. In other words, an increase in the guaranteed rate would decrease the value of the investment fund, expenses, and *muḍārabah* contribution. Further, in panel 1B the volatility parameter value is changed to $\sigma = 0.20$; this was employed to test the influence on the change of the volatility parameter to the value of the investment fund, expenses, and *muḍārabah* contribution. By using $\sigma = 0.20$, at $g = 0.000$, we found that the value of investment increased from $86.86 to $105.37, and the value of expenses rose from $28.87 to 30.06, while *muḍārabah* contribution also rose from $115.72 to $135.43. This indicates that the volatility of reference fund has a positive effect on the value of investment fund, expense, and *muḍārabah* contribution. It means that a change in the value of the volatility parameter would increase the investment fund, expenses, and contribution of *muḍārabah*.

**Table (7) The value of investment fund, expenses and *muḍārabah* contribution in the Equity-linked policy with the finite time horizon *T* = 5 years**

| Panels | Change of the Variable | Value of the Investment Fund (Equation 31) | Value of the Expenses (Equation 32) | Value of the *Muḍārabah* Contribution (Equation 33) |
|---|---|---|---|---|
| Panel 1 A | $g = 0.000$ | 86.86 | 28.87 | 115.72 |
| $Rp = 0.035$ | $g = 0.003$ | 84.74 | 22.83 | 107.57 |
| $\sigma = 0.15$ | $g = 0.007$ | 81.94 | 14.42 | 96.36 |
| Panel 1 B | $g = 0.000$ | 105.37 | 30.06 | 135.43 |
| $Rp = 0.035$ | $g = 0.003$ | 103.46 | 24.45 | 127.91 |
| $\sigma = 0.20$ | $g = 0.007$ | 100.92 | 16.93 | 117.86 |
| Panel 2 A | $g = 0.000$ | 95.27 | 34.20 | 129.47 |
| $Rp = 0.050$ | $g = 0.003$ | 93.12 | 28.80 | 121.92 |
| $\sigma = 0.15$ | $g = 0.007$ | 90.26 | 21.46 | 111.72 |
| Panel 2 B | $g = 0.000$ | 112.13 | 35.82 | 147.95 |
| $Rp = 0.050$ | $g = 0.003$ | 110.21 | 30.60 | 140.81 |
| $\sigma = 0.20$ | $g = 0.007$ | 107.66 | 23.57 | 131.24 |
| Panel 3 A | $g = 0.000$ | 106.50 | 39.92 | 146.42 |
| $Rp = 0.070$ | $g = 0.003$ | 104.34 | 35.23 | 139.57 |
| $\sigma = 0.15$ | $g = 0.007$ | 101.46 | 28.86 | 130.31 |
| Panel 3 B | $g = 0.000$ | 121.05 | 42.50 | 163.56 |
| $Rp = 0.070$ | $g = 0.003$ | 119.14 | 37.84 | 156.98 |
| $\sigma = 0.20$ | $g = 0.007$ | 116.60 | 31.54 | 148.14 |

Further, in the second panel, we change the appropriate probability return of the seller $Rp = 0.050$ (see panel 2A), the volatility is set equal to $\sigma = 0.15$, and the guaranteed rate is $g = 0.000$. We find that the value of the investment fund is \$95.27, the expense value is \$34.20, and the value of *muḍārabah* including the expenses is \$129.47. Then, if the variable guarantee rate changes to $g = 0.003$, the value of the investment fund decreases to \$93.12, the expense value becomes \$28.80, and the value of *muḍārabah* including expenses becomes \$121.92. Further, when the guaranteed rate is changed to $g = 0.007$, the value of the investment fund also decreases from \$93.12 to \$90.26, the expense value becomes \$21.46 and the value of *muḍārabah* including the expenses becomes \$111.72. The change of the guaranteed rate (increase) affects the value of the investment fund, expenses, and *muḍārabah* contribution (decrease). At the panel 2B, the change of volatility parameter, $\sigma = 0.20$ is carried out to test the effect of the change a volatility parameter to the value of the investment fund, expenses, and *muḍārabah* contribution. By using $\sigma = 0.20$, at $g = 0.000$, we found that the value of the investment increased from \$95.27 to \$112.13, and the value of expenses rose from \$34.20 to \$35.82, and *muḍārabah* contribution also rose from \$129.47 to \$147.95. This indicates that the volatility of reference fund has a positive effect on the value of the investment fund, expense, and *muḍārabah* contribution. It means that a change of value in a volatility parameter would result in an increase in the value of the investment fund, expenses, and contribution of *muḍārabah*. Then, the important conclusion is that an increase in the appropriate probability return of the seller and volatility affects the value of the investment fund, expenses, and *muḍārabah* contribution.

In this setting, the periodical premium can be determined with the European call option embedded in the policy value at the maturity of the insurance contract evaluated by the Black-Scholes model and the dynamic Escrowed model. Using equations (24) and (25), the result can be seen as below:

**Table (8) The periodical premium should be paid by policyholder at the inception of equity-linked policy at the finite time horizon $T$ = 5 years**

| Panels | Changeof theVariable | Value of the Periodical Premium (Equation 29) |
|---|---|---|
| Panel 1 A | $g$ = 0.000 | 130.11 |
| Rp = 0.035 | $g$ = 0.003 | 122.09 |
| σ = 0.15 | $g$ = 0.007 | 111.06 |
| Panel 1 B | $g$ = 0.000 | 149.82 |
| Rp = 0.035 | $g$ = 0.003 | 142.43 |
| σ = 0.20 | $g$ = 0.007 | 132.55 |
| Panel 2 A | $g$ = 0.000 | 143.85 |
| Rp = 0.050 | $g$ = 0.003 | 136.44 |
| σ = 0.15 | $g$ = 0.007 | 126.42 |
| Panel 2 B | $g$ = 0.000 | 162.34 |
| Rp = 0.050 | $g$ = 0.003 | 155.33 |
| σ = 0.20 | $g$ = 0.007 | 145.93 |
| Panel 3 A | $g$ = 0.000 | 160.81 |
| Rp = 0.070 | $g$ = 0.003 | 154.09 |
| σ = 0.15 | $g$ = 0.007 | 145.01 |
| Panel 3 B | $g$ = 0.000 | 177.95 |
| Rp = 0.070 | $g$ = 0.003 | 171.50 |
| σ = 0.20 | $g$ = 0.007 | 162.83 |

In Table 8, we also use two panels, with every panel divided into two parts. In panel 1A, by using the appropriate probability of return for the seller, $Rp$ = 0.035, the volatility of the reference fund $\sigma$ = 0.15 and the guaranteed rate of $g$ = 0.000, we obtain the periodical premium paid by the policyholder of $130.11. Assuming the guarantee rate increases to $g$ = 0.003, the change of variable guarantee rate has a negative effect on the periodical premium. The same is the case when the variable guarantee rate is changed to, $g$ = 0.007. At the same time, we assume a change of the volatility value and set $\sigma$ = 0.20, and $g$ =0.000 (see panel 1B). The periodical premium increases from $130.11 to $149.82. Based on this result, we find the change in volatility has a positive effect on the periodical premium payment paid by the policyholder. In this case, the influence of changing the variable guaranteed rate and volatility is the same for panels 2A and 2B. Further, in the setting, we tested a comparison of three panels applied to calculate periodical premium, with the assumption that there is a change in the appropriate probability return of the seller, $Rp$ (see panel 1 $Rp$ = 0.035, and panel 2 $Rp$ = 0.050). We found that increasing the parameter of appropriate probability return of the seller positively influenced the periodical premium paid by the policyholder at the inception of the equity-linked policy at the finite time horizon $T$ = 5 years.

## 5. Summary and Conclusion

We develop a new modeling of premiums that considers the properties of family *takāful* contracts. A mortality derivative has been created to avoid the hidden elements that still exist in the previous formula adopted and applied by *takāful* insurance companies to calculate mortality risk. In the setting, we focus on the model of the premium for an equity-linked policy (*unit-linked product*) and adopt the option pricing model of Black-Scholes specifically to determine fair periodical premiums, taking into account minimum death benefits, the value of surrender options, and maturity guaranteed payoffs. The analysis uses numerical simulation, performed with MATLAB. We also added various assumptions for the underlying asset price following negative discrete dividend extended from the dynamic escrowed model (Bos et al., 2003; Costabile, 2013), which was repurposed to obtain an unbiased option price for the underlying asset and to redefine the practice of interest and risk-free rate with the appropriate probability cost of the seller (Vogel & Hayes, 1998).

In the present paper, we find that the death benefits planned by the policyholder affects the investment fund, periodical *tabarruʿ*, and total periodical premium paid annually at the beginning of the year. Furthermore, the value of the surrender option obtained from the payment of insurance periodically reduces expenses such as the *tabarruʿ* premium and operational costs. It is clear that the surrender option value received is the investment fund value. Besides that, the payoff of the maturity benefits for the policyholder at the maturity of the contract is shown by the guaranteed payoff. For instance, if the guaranteed rate is set equal to $g$ = 0.000, then the policyholder would receive a payoff from the insurance company at the maturity of the contract equaling the periodical reference fund, i.e. \$500. Then, if there is a change in the variable guaranteed rate, say to $g$ = 0.002, the amount received by the policyholder at the maturity of the contract would increase to \$503.01 (see, Table 1). We demonstrate a satisfactory product with a fair periodical premium that has unlimited flexibility in product features and fulfills Sharīʿah compliance criteria comprehensively.

## 6. Limitation and Research Contribution

In general, this study is conducted from a viewpoint of Islamic insurance (*takāful*). The study develops a theoretical model for the premium and demand for family *takāful*. The aim of the study is twofold: Firstly, we develop a new model for premium payments on family *takāful* specifically related to equity-linked policy products. We use stop-loss and continuous-time contracts and comprehensively consider the properties of *takāful* contracts namely, *tabarruʿ* and *muḍārabah* contracts. It is intended to ensure a new product that could be attractive to customers interested in purchasing *takāful* products. Further, the findings relating to modeling for the premium of *takāful* life insurance can provide several contributions for the practice of *takāful*. First, the new modeling approach for the premium that has been created and developed is capable of being an alternative for *takāful* operators to change the current model that has been implemented to determine life insurance premiums. We design a new model specifically for equity-linked policies (known as unit-linked products). Our new product provides mutual benefits for both parties – the *takāful* operator and the policyholder – and overcomes the limitations associated with traditional *takāful* products.

Second, in the new model that has been created and developed, we consider the broad properties of *takāful* contracts (based on our view that several elements are still unclear in the practice of *takāful* life insurance). In particular, we discuss in detail issues surrounding excess of premium and *tabarruʿ* and also the surplus sharing of *tabarruʿ* and *muḍārabah*. Thus, we create and develop a new approach and mechanism to address some of these issues. The new model is capable of being a guideline for *takāful* operators to eliminate problems in traditional contracts as well as offer insurance products more profitable for both parties. Finally, our model can be used as a guide for government to determine a regulatory framework for the minimum guaranteed rate that should be included in the calculation of premium. It is important to ensure that premiums paid by policyholders are low (cheap) and thus increase the demand for Islamic insurance services.

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# نمذجة قسط وخصائص عقد التكافل الأسري (التأمين الإسلامي على الحياة)

**جوماديل سابوترا، سوهال كوسايري ونور أزورا سنوسي**

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**المستخلص**. يعتبر قسط التأمين وظيفة حتمية للتعويض عن الخسائر الناجمة عن الأحداث العشوائية والغير متوقعة، وعنصرا هاما في عمليات شركات التأمين، وتركز هذه الورقة على ممارسة التكافل الأسري، وتنظر في خصائص العقد في تحديد أقساط التأمين بشكل شامل وبطريقة تجعلها متوافقة مع الشريعة الإسلامية. في هذا الصدد قمنا بتطوير معادلة جديدة لمشتقات الوفيات ونموذجا للقسط الخاص بوثيقة مرتبطة بالأسهم (منتج مرتبط بالوحدات). وقمنا كذلك بتكييف تسعير الخيارات لنموذج بلاك سكولز (Black-Scholes model) للأقساط الدورية، مع مراعاة الحد الأدنى لاستحقاقات الوفاة، وقيمة خيار التسليم، وضمان الدفع عند نهاية العقد. كما أضفنا بعض الافتراضات لأسعار الأصول الأساسية بإتباع امتدادات توزيع أرباح منفصلة سلبية من النموذج الديناميكي المضمون، والغرض منه هو الحصول على سعر الخيار غير متحيز للأصل الأساسي. وقد توصلت النتائج أن منتج مرضي مع قسط دوري عادل، والذي فيه مرونة كبيرة في معالمه ويتوافق بشكل شامل مع أحكام الشريعة الإسلامية.

**الكلمات الرئيسية**: قسط التكافل العائلي، السياسة المرتبطة بالأسهم وأنموذج توزيع الأرباح الديناميكية المنفصلة.