THEORY OF GROWTH OF A FIRM IN A ZERO INTEREST RATE ECONOMY

BY

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DEDICATION

TO MY SISTER

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FOREWORD

I am deeply gratified to write a foreword to this monograph on the micro-economic implications of the elimination of interest for the growth of a firm. The fact that an M.I.T. trained non-Muslim professional economist found the subject interesting enough to devote his energies to it is an indication of the wider attention some of the basic ideas in Islamic economics, such as interest-free financing, are now receiving.

Dr. Mukherji has followed in the present monograph the same methodology as in his earlier paper “A Micro-Model of the Islamic Tax System” *(Indian Economic Review,* Vol. XV, No. 1) where he took as given, the Islamic system position on a certain issue, and analyzed in detail some of its economic implications. In preparing for this monograph, the Centre was glad to provide Dr. Mukherji with relevant Islamic economics literature and a list of system’s “parameters”, and as usual, with the referees’ reports and our Academic Committee’s notes. The choice of assumptions, method of analysis and the ensuing conclusions are the author’s own.

Dr. Mukherji has demonstrated one way of analyzing the workings of an interest-free system, and has provided some specific conclusions. As in much of economic theory, not all the conclusions are definitive. This does not detract from the value of this work, which is an important first step into a new field.

We look forward to further contributions on this fine from Dr. Mukherji and other professional economists.

Omar Z. Hafiz
Director

PREFACE

This is a monograph with a single focus, namely, the theoretical problems concerning the growth and investment of a firm in an Islamic economy. The only aspect of such an economy that has been studied here is the consequence of removing the conventional money market and loan transactions at non-zero interest rates. Clearly this is a partial view of the Islamic economy. In particular, the profound problems of institutional change and reform and the macroeconomic problems concerning taxation and social security that I have analyzed in an earlier paper* have not been included in the discussion. This is not to deny the importance of these problems; it simply reflects the belief that unless the building blocs are first sorted out nothing substantial can be constructed.

I have tried to keep the algebra to a minimum and make do with geometry as far as possible. This procedure has its limitations but it seemed, as the study progressed, that the essential features that appear - like certain non convexities in feasible regions or multiple equilibria - could be analyzed adequately by geometry. In particular, one pleasing aspect of the model was its ability to generate interior solutions in the presence of scale economies and the geometry was adequate to demonstrate it. And, when all is said and done, the lack of a concrete real life point of reference makes too vigorous a modelling slightly risky. Even so, the constraint of space has at times made the exposition terse. Not, we hope, unduly so.

I have many debts to acknowledge Prof. K.A. Naqvi of the Delhi School of Economics introduced me to Islamic economics. The Director of the Institute of Islamic Studies, Tughlaqabad, Delhi, and his fine team of research scholars have been deeply involved in Islamic economics for several years now. Their monthly seminars helped me both to get a feel for the subject and to overcome the otherwise insuperable barriers of language and ignorance of the Islamic scriptures. I thank all these people.

On the material presented, the criticism and suggestions of Jacque Dreze of CORE, Belgium, were of great help and this monograph owes a lot to him. A.M. Khusro and Mehfooz Ahmed made valuable comments during a Tughlaqabad seminar in Summer, 1980. Two anonymous referees pointed out many errors of presentation and ambiguities and suggested several points of substantive improvement on an earlier draft. I am thankful to all these people. And last but most of all, as any reader even without much patience will observe, my intellectual debt to Adrian Wood's work is massive. It is a pleasure to acknowledge it here.

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* Mukherji (12)

It was observed by one of the referees that there is no firm 'conclusion' in the monograph. I plead guilty. I have tried to provide a methodology, a structure of analysis, and derived some of what seemed to be the more interesting propositions. But like any partial equilibrium model, ours will need a lot more work and extension to a general equilibrium model before robust conclusions can be supported.

Islam is a living philosophy followed by numerous and diverse peoples of the world. If the monograph seems cold to the reader I apologize, but assure him or her that there was no lack of excitement in the work involved, the effort spread out over several years.

The theory of the Islamic economy is being developed largely ahead of its practice. That ought to lessen the burden somewhat when I claim the responsibility of all the errors that remain.

January, 1984.

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1. INTRODUCTION

I.1. In recent years a lot of very lively research has been undertaken on the qualitative properties of economic systems that seek to follow the precepts of the Holy Qur'an. Dr. M.N. Siddiqi's authoritative *Survey of Muslim Economic Thinking* runs into seven hundred items, mostly of recent vintage. This research gains great practical significance in view of the desire of a whole range of Muslim countries to actually run their economies according to the precepts of the Holy Book. And, as is well known, one of the precepts to draw the most attention has been the great injunction banning the giving or taking of interest in loan transaction. Quite naturally, the question has been raised as to whether or not interest is a *necessary* adjunct of a modem money market. A wide range of enquiries are opened up by this question like the nature of money, of financial institutions and banks, the role played by the rate of interest in various economic systems and so forth. The nature of interest, its explanation and its determination constitute some of the most controversial and complicated problems in economic theory but that does not make the study of a 'no-interest' system fanciful; one has just to look at the literature to see how often it has come back to the question of "Can the equilibrium interest rate be zero?" To complicate matters further, these problems on the descriptive/explanatory side are matched by an equally complex set of issues on the normative side. Does the institution of interest-paying and receiving commercial banks help or hinder the system in attaining a socially desirable allocation? Is speculative demand for money disruptive? What is the 'optimum' quantity of money and the 'optimum' rate of interest? These are some of the issues that naturally crop up in this context.

Without in anyway denying the importance of such problems, we are going to analyze a somewhat different one, and it may be best to clearly define the latter at the very outset, since it is not what is usually presumed to be the object of discussion in the "interest problem" of Islamic economics. We shall try to study the partial equilibrium problem of the growth and investment of a firm which has to operate in an economy in which, following the precept of Islam, interest-bearing loans no longer exist - neither to give, nor to receive. Observe that while studying a firm, we are involved with the *previous* aspect; a firm *pays* interest while borrowing, while much of the literature as far as we could ascertain has really worried with the latter aspect, i.e. the problem created by not being able to lend money and *receive* interest.

I.2. Two important qualifications to our project have to be made. First, we are *not* going to study the Walrasian general equilibrium model under a new restriction that the interest rate is zero. To be sure, the latter can be studied, and it mayor may not yield interesting results; but we are not doing it here. The crux of the matter is that not just a price (of money) is being exogenously specified in ZIRE* but a major component of the Walrasian market system - the capital market - is going. There is really no 'business loan' in the conventional sense anymore. From well known analyses which need not be repeated here, it is known that the absence of such a market will almost surely make the attainment of the earlier type of a (competitive) market equilibrium an impossibility. Knowing this, we make no attempt whatever to use or rely on the traditional marginal value product or marginal rate of substitution arguments in the description of the equilibrium. While the loss of well known descriptive formulae increase the technical problems, it has its uses, especially as during the course of analysis significant non-convexities appear in several important feasible sets. These would seriously jeopardize the characterization of equilibrium if we were wedded to the *competitive model,* but turn out to be tractable when no such restriction is imposed[1]. It would be in order to invite the attention of the reader to this aspect of the analysis, otherwise one might be left wondering how, in spite of those non-convexities, comparative static results are being churned out.

In this connection, we might point out an important analogy-the problem of income distribution. It is well known that in the absence of uncertainty and market imperfections the Walrasian equilibrium has several “efficiency” properties but, theoretically, it leaves the resulting income distribution free to take on any shape depending on the equilibrium. If, now, an alternative economic system is proposed which starts with the axiom that the income distribution is given exogenously, by reference to the desires of the society, then the new system is likely to depart from some - if not all - of the equilibrium properties of the earlier one and it is beside the point - if not a bit illogical to try prove the contrary or to ‘prove’ that the new system is 'better' than the earlier one. The latter judgement has already been made - while specifying the income distribution - and cannot be ‘proved’ afresh; its consequences can only be illustrated by the resultant changes in the equilibrium.

I.3. The second point, coincidentally enough, happens also to be a disclaimer. We are *not* going to study here the institutional problems of a no interest rate - system, or that of a conventional economic system making a transition to a no-interest rate - system. Needless to say, the choice involves no value-judgement whatsoever; if anything, in my judgement, the institutional problems will be of paramount importance to policy-makers for a long time to come, and may indeed be the heart of the study

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* Zero Interest Rate Economy.

1. See 1.7 and Chapter IV below.

of an Islamic economy. I share the guilt of a lot of theories in attacking first a much smaller and easier-to-manage problems. Many a time in the text we talk of economic consequences of parameter or policy changes without any reference to social welfare (social justice, *al 'Adl)* and the reason simply is that institutional reform has been already subsumed and we are providing no arguments what soever for or against it. In other words, the results presented have no bearing on

(i) how to effect institutional reform or
(ii) whether the reforms are justified on some basis or the other.

If this seems to leave only a narrow field for us, it is best that we register it at the outset; there does not seem to be any special reason why an economist as an economist - can have anything special to say on those matters.

I.4. The same desire to restrict ourselves to a specific, partial equilibrium problem lead us to avoid bringing in the money-market directly into the picture. This is a separate problem, an important one, and has been justifiably drawing a lot of attention; in particular, the role of speculative demand in the money market and the question whether by preventing commercial loans the system becomes more or less stable are being investigated. The analysis of the money market also links up pretty smoothly with institutional reform in banking and other financial intermediaries in an Islamic economy. But, for the theory of the firm, all this is part of the environment. It is our impression that it is as yet too early to develop a full general equilibrium model.

Whether individuals invest their savings in firms or deposit them in banks, the yield will be an uncertain dividend (paid from the profits of those firms or banks) and the equilibrium in a multi-asset model would at most require that the expected dividends across assets are equal, *but not necessarily so,* because the model is not competitive. As in any partial equilibrium model, we implicitly assume that there is one bank (or firm) paying one dividend. It would be the task *of* the monetary theorist to study and explain the nature and spread of dividend rates that *the banking system as a whole would sustain*[2]. For our purpose, the critical point is the replacement of a known, non stochastic interest rate by an uncertain rate of dividend payment; of a sure source of funds (credit) by an unsure one (equity investment). This is the problem discussed at length in this monograph.

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2. In a competitive asset market, the own rates are equal across assets in equilibrium. See Bliss (3)

But we do not need to take any position at this stage on the role of the banking sector or its relationship to industry in the changed environment, in the belief that the inner mechanism of firm behaviour must be understood first before one can investigate the nature of its interaction with external agents. In particular, one result of traditional economics becomes irrelevant, which is the equalization of own rates of different assets in competitive equilibrium. We no longer can maintain the assumption of competition; in fact we devote some time to study what is meant by “more or less competition” in ZIRE (Ch. V below).

I.5. In Chapter II is provided a brief survey of the theory of profits and investment in a firm. As is well known, several alternative theories and descriptions of the capital market are available but one has to be careful about which one to choose as a starting point because it must be ensured that the assumptions of the chosen theory are consistent with the specification that our model requires. Thus, the competitive model a la Fisher or its analogue in the capital market (the Modigliani-Miller theorem) cannot be used because they are seen to assume away the problem which we are trying to solve.

It is argued that the model of investment and financing developed by O. Williamson (16), Adrian Wood (17) and others is one in which the source of finance for investment does make a difference to the equilibrium of the firm - often a substantial one. Wood’s model, in particular, is chosen as our point of departure because of its clarity and ease of manipulation. As it stands, however, it does not easily accommodate some of the specific questions that we might be interested in. In Chapter III, therefore, we give a statement of Wood’s model and suggest certain modifications/extensions relevant for our purpose later in the text. The thrust of the discussion is to show how financial constraints on the one hand and market forces, nature of competition and technology on the other determine the profit margin and rate of growth (of size, output) of firms in an essentially oligopolistic industry.

I.6. In Chapter IV we construct the model of what we call the firm in the Zero-Interest-Rate Economy by leaving other things the same as in Wood’s model but driving out interest-bearing loans from the financial side. In other words, of the three sources of funds in Wood's model - borrowings, undistributed profits and equity investment - only the last two now remain and what is often the largest source of short-run investment and working capital, i.e., borrowings, are no longer available. The crux of the matter is that when borrowings are replaced by equity financing, a significant change is introduced in the supply side of funds. Even though it is true that borrowers may not always get the kind and volume of loans that they want exactly, as a start it is not a bad assumption to make that given the terms of loan stipulated by the lender (often on a negotiated basis) the quantity of loan is decided upon by the borrower. But in equity financing the roles get reversed; it is the borrower, the firm, that has more control over the terms and conditions while the lender, the investing public, decides upon the quantity of investment to make. The comparison of the end result for the volume of funds available to the firm is not easy, not even if one made strong assumptions like the expected rate of dividends equal the rate of interest (an assumption that we do not make). We, therefore, resort to a complete analysis of all the cases they being distinguished by whether the maximum amount of equity financing is greater than, equal to or less than the quantum of borrowing that was being made previously by the firm. It is shown that significant non-convexities emerge in the feasible sets, that these can be handled because we are not committed to the assumptions of "competitive profit maximization", in which case they could cause serious problems in the analysis of equilibrium and comparative statics, and finally, that in what seems to us to be the more interesting cases, the equilibrium of the ZIRE firm will demonstrate higher growth and lower profit rates as compared to those of a capitalist firm.

I.7. We conclude this Chapter with an interesting exercise that suggests why capitalist firms face difficulties in handling scale economies, how the latter can be theoretically better analyzed in a model that includes the financial constraints and that the ZIRE firm might be in a better position to exploit these economies.

I.8. An important set of questions regarding the structure of the industry are taken up in Chapter V, those relating to the implication for the equilibrium of more or less intense competition from rival firms. We discuss the impact of increasing competition on the ZIRE firm and contrast it with the capitalistic case. Of course, in the limit, if there is an infinite number of rival firms, then both the systems end up in a zero-profit, zero-growth situation. But the interesting contrasts are provided by the approach to this limit, when the number of competing firms is finite and rising, and capitalist and ZIRE industries are seen to adjust very differently.

I.9. In Chapter VI we take up the important question of strategic behaviour and uncertainty. This is a vast and complicated area and in one respect unmanageable within the bounds of our monograph - it becomes algebraically complicated if a full analysis is attempted. All through the text we have argued that replacement of credit by equity financing makes the model stochastic, but anybody can see that explicit random terms and noise variables have not been introduced. In other words, our modelling tries to use the non-stochastic structure of analysis since it is simpler and well understood[3]. (Footnote 3 see on next page)

But this gambit fails while directly confronting questions of uncertainty. To be sure, strategic behaviour and game problems have often been analyzed by non-stochastic models. Observe the literature on Cournot duopoly. But increasingly it is felt that in such problems it is best to have an explicitly stochastic model.

In this Chapter, therefore, we indicate the issues, point out the kind of modifications that would have to be made and the essential properties that the new structure would have to have, leaving explicit algebraic formulation for a later occasion. We simply try and underline certain kinds of problems that are likely to surface for the ZIRE firms. Then we try and recast two explicit models of oligopoly (Coumot's and Solow's) in terms of our model and analyze the consequences of imposing the financial constraint on them. The sharp difference of results brings out the importance played by behavioural assumptions in oligopoly and by the financial constraint in a problem of limit pricing.

I.10. Finally, since we feel what we have developed here following on Wood's work is essentially a methodology, rather than a set of propositions, we try and give an example of the use of the methodology in Ch. VII. The model developed in it seeks to capture some economic issues facing extractive industries. This is a problem that has attracted a lot of attention in the last decade but to the best of our knowledge no researcher has investigated the role played by financial constraints in this problem. Hence, even on its own, it has some interest. In addition we show that the equilibrium of the firm in ZIRE would be quite distinct and, in particular, encounter a striking possibility of a pure conservationist equilibrium with a negative growth rate which, in terms of our model, is unattainable by a capitalist firm.

I.11. In conclusion we make an attempt to round up the discussion, collect together the major results, suggest the most obvious extensions and underline some short-comings of our analysis.

I.12. Modelling ahead of experience is a risky exercise and yet, at times, the intellectual challenge of an exciting new problem proves to be too alluring. And often the investigation of a new system yields interesting insights into the properties of the system being altered. It is in that spirit that the results of this monograph are being offered for public scrutiny.

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3. This is not unusual. The entire corpus of standard macro-economic theory treats liquidity preference in this manner.

II. PROFITS AND INVESTMENTS IN A FIRM: A BRIEF SURVEY[4]

II.1. We begin with the problem of model selection for the Zero-Interest Rate Economy. The choice of a model for analyzing a particular problem is often half the job. It largely determines the kind of specific questions that can be asked, in addition to committing the analyst to some pretty strong positions on what the underlying structure of the economy looks like. The present problem is a good example.

II.2. For example, one could cast the problem of the firm in ZIRE in terms of a competitive profit maximizing firm which is debarred from borrowing funds to finance investment. In such a world, financing of investment is a portfolio problem - it in no way decides the kind or volume of investment. In particular, we have the Modigliani-Miller theorem (11) which proves that the ratio of Debt to Equity of a firm leaves unaffected the valuation of the firm as well as its ranking of investment projects. Hence maximizing either current profits or present discounted value of the stream of future profits will be left unchanged by any specific value of the Debt-Equity ratio, including zero. Both current output (and price) and future growth (and investment) will be unaffected.

What will happen, however, is a sharp rearrangement in both savers' and investors' portfolios. In particular, if prices are steady, the future is known with certainty and the money market is competitive then all bonds earn a constant, real interest rate which, to the saver, is a sure return and to the investor, a fixed, contractual cost obligation. But its disappearance makes no difference to such a scene of tranquility since in a steady state in a competitive economy all assets will be earning the same own-rate [see Bliss (3)].

II.3. It *will* make a difference, however, if the assumption of full certainty is dropped. If the future course of inflation is uncertain then the real rate of interest on bonds is itself uncertain since the real rate is the money rate adjusted for inflation. Hence all assets will have uncertain returns and the firm's costs will have a random component. In fact if the firm is following a stable nominal dividends policy then inflation at an uncertain rate randomizes all costs.

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4. This is not a survey of the massive literature on investment. Rather, we seek to provide the rudiments of a theory of investment in which the financial constraint is active, contrast it with the traditional theory and suggest reasons why we use the former in the analysis.

II.4. It seemed to us more fruitful *not* to pose the problem in the neoclassical mold since financing is a secondary issue in it. The really interesting question, in our opinion, is what will happen in an economy in which financing *does* affect the growth and profits of firms. We need, therefore, to first describe the bare outlines of such a model before introducing the modification due to the absence of interest.

II.5. There is a long heritage of the theory that postulates that real world firms do not maximize short-run profits. Beginning with Kalecki's monopolists (7) and the full-cost-pricing firms of the Oxford study of Hall and Hitch (5), it has been variously postulated that modern markets are basically run by a cost plus mark-up pricing formula. Massive factual information has also been accumulated to back up this position in recent times (Star buck (14)) along with more comprehensive theorizing by economists like Baumol (1) Williamson (16) and Marris (9). The last two explicitly tried to accommodate the role of the manager in a modern corporation, owned in theory by shareholders who receive a (random) dividend and have no way of enforcing a maximum-profit objective.

II.6. In all this, however, attention was kept focused on the input-product sales-profit dimensions and little effort was spent on integrating the role of the financial side of the firm's activities and its current decisions. Although as early as 1932, Berle and Means (2) had concluded from their massive study of U.S. Corporations that the advent of the share market had made a fundamental difference to the operations of the firm, no theory of the corporate firm separate from the traditional imperfect competition was fully worked out. The major reason, one suspects, is that when focusing on short run equilibrium the question about the source of investment financing can be shelved. In addition, in a Fisherian world, it makes no difference to the maximization of the discounted sum of a stream of profits as to how the growth of the firm is financed. Modigliani and Miller (op. cit) seek to trans late this position in modern day terminology by showing that under their assumptions the Debt/Equity ratio is immaterial for either a valuation of the firm or for the ranking of investment projects. How much of gross profits should the firm reinvest? The answer is provided by the rate of interest. The firm should reinvest upto the point where the rate of return on the marginal investment equals the interest rate. The retention decision being given, the rest of profits are paid out as dividends thereby fixing the latter. If this makes the shareholder unhappy then he can sell off his shares and buy some other. In other words, he can "declare his own dividends" as it were. The joint action of a firm to reinvest (which raises the value of its shares) and the shareholder to sell (which depresses the value of the shares) determines the price per share and the equilibrium dividend rate across the market.

II.7. This theory collapses on two counts. From the firm's point of view, there is ample empirical evidence that firms do not use an external standard of profitability like the interest rate to decide upon the rate of retention of gross profits (Kuh (8), Meyer and Kuh (to), Turnovsky (15) etc.). Rather, it is the dividends payments policy which is the independent variable[5]. Analytically, even within the neoclassical p.d.v. (present discounted value) maximizing structure, Solow (13) has shown that firms with very different sizes and rates of growth can earn the same maximum present discounted value so that this criterion fails to tell them apart whereas such different firms are likely to have radically different dividends policies.

II.8. More seriously, from the householder's point of view, continuously adjusting his portfolio by buying and selling shares to maintain his dividends is an unrealistic assumption on two counts. First, there are very substantial transaction costs in buying and selling shares, particularly if the amounts involved are small as is likely to be the case for most individuals. Quite apart from brokers' fees and paper work, just to have enough information is more than a full time job for most people. What compounds this is the second reason that the market for shares is the most volatile of all markets, perpetually under speculative pressures. This on the one hand makes it impossible for individuals to freely buy and sell without serious risks of capital loss and on the other hand prevents any systematic influence of retained earnings or investment on share values from being realized. For after all the share market trades titles to capital assets which themselves are very hard to resell and are mostly product specific. Those capital assets have hardly any forward markets to speak of so that the entire brunt of the uncertainty created by the absence of those forward markets is borne by the share market. For most shareholders the liquidity gain of the dividends far exceeds the possible future potentials of retained earnings in the company whose shares they hold. It is the dividends payments policy of the firm which thus becomes the key variable of the problem.

II.9. The previous analysis in very much more detail is developed by Wood (17) who then goes on to construct a model of equilibrium growth of a firm. We intend to use it after suitable modifications to outline a theory of the firm in ZIRE. The advantages are obvious. First, Wood's model is sufficiently down to earth to get a bite on the problem of investment in a firm. Secondly, since the loan market plays a central role in his work (as it ought to) but is absent in ZIRE, we get a sufficiently sharp contrast to highlight the differences adequately.

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5. This will be of critical importance in our discussion of ZIRE below.

III. ADRIAN WOOD'S ANALYSIS - REVIEW AND EXTENSION

III.1. The theory we are about to describe assumes that the objective of the firm is to achieve the highest possible growth of sales revenue subject to three constraints. Those are on (a) the growth of demand (b) the growth of capacity and (c) the availability of finance for investment.

It should be noted that sales revenue maximization and profit maximization amount to the same short-run price-output decision if the average costs are constant. The price of the product is then set by a proportional mark-up on unit costs and the mark-up decides the profit margin. However, the two theories begin to diverge when profits net of dividends are considered to be the main source of reinvestible funds and investment is taken as the source of capacity expansion. At the same time part of the investment is devoted to sales expenditure which, along with exogenous factors, determines the growth of demand. The joint action of capacity expansion and demand-growth determine the limits of sales expansion. The finance constraint then is used to find out the maximum achievable rate of growth of sales.

The implication of this scenario is that it drives a sharp wedge between static and dynamic analysis by specifying the source of investible funds. It is here that the finance constraint comes into play, unlike in the static equilibrium, whether profit or sales-maximizing, in which the implicit assumption of instantaniety permits one to neglect the fact that outlays have to be made *before,* often very much before revenues start to flow in. The changes wrought by the finance constraint are often dramatic. As we shall show, the famous problem-cases of increasing returns or economies of scale, for example, can be handled quite adequately.

In the subsequent analysis we retain Wood's notation in order to facilitate comparison.

III.2. The Opportunity Set: Concentrating first on the real variables, the opportunities open to a firm can be summarized by the following relation which Wood calls the opportunity frontier:

$$\pi \leq \mu\,(g, k) \qquad \ldots\ldots(1)$$

where

π = the profit margin, P/V

P = profits

V = sales revenue

g = the proportional growth in sales revenue (V – V) / V, V being the exogeneously given past level of sales, may be yesterday's.

k = the 'investment coefficient', I/(V – V) where I is the volume of investment; this is simply gross capital output ratio.

III.3. Equation (1) summarizes, for given g and k, the level of π that is feasible. Its two major properties are as follows:

(a) The opportunity set is bounded. This is obvious; there are upper limits to the amount that the firm can sell, the profits it can make and, if it is to survive, lower limits to the amount it needs to invest.

(b) We assume that the growth of aggregate demand is exogenously given; both for the economy and for the industry. The firm's demand grows depending on its sales policy, unit cost (net of sales cost) and product mix. But sales cost is a two-edged weapon; it raises demand but, cet. par., lowers profit margin. Beyond a certain point, growth of sales can be achieved only at the cost of lower profit margins. Given $k = k_1$, the curve μ is thus expected to have the following general shape. (Fig. 1):

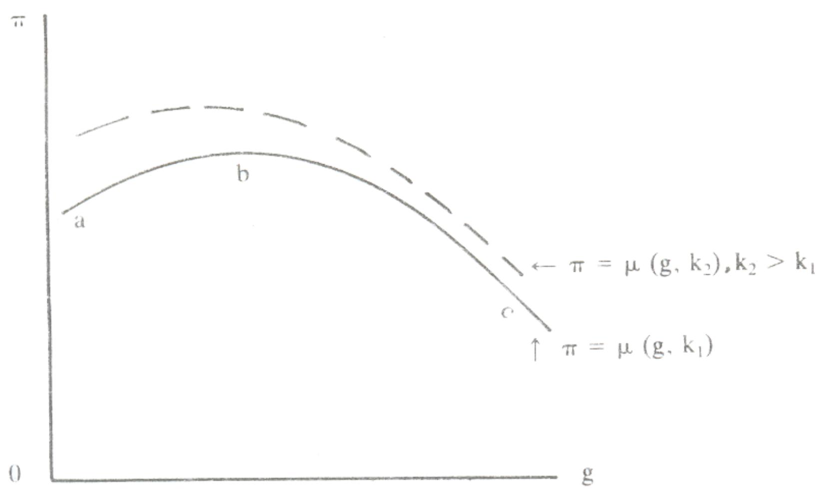

Fig. 1

III.4. Over the range ab in the unbroken curve extra sales expenditure succeeds in pushing up demand faster than it raises prices. Hence both g and π increase. But this trend cannot last forever and it would be inefficient for a firm to stop before b. The real trade-off between g-π is over the range bc. The simplest impact of an increase in k on 'TT is to assume that at all values of g, a larger capital stock permits lower unit costs through extra efficiency and hence it raises π.

III.5. A more complicated story is to postulate that a higher k raises π only after a minimum size of plant has been achieved – a more Marshallian kind of assumption. In that case a rise in I while raising g will lower π for a while before the economies of scale and technical efficiency are felt. Conversely, on a higher k-basis, the range of both π and g jointly rising will persist for longer as compared with a lower k-base (Fig. 2).

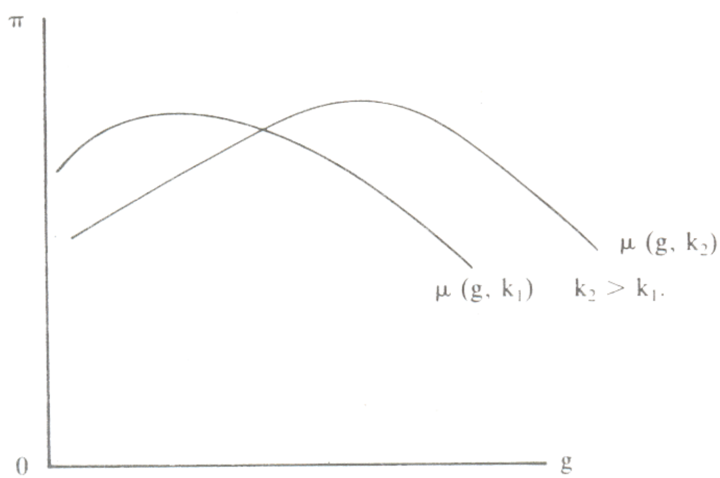

Fig. 2

Wood considers only fig. 1 in his model. In our judgment that would be tantamount to identifying higher k with technical progress. It is the latter that unequivocally raises π at all g. But since higher capital output ratios imply a newer technology usually by way of new and more expensive machines, we prefer to use our somewhat more general presentation of Fig. 2[6]. This is our first modification of Wood's model, one which ought

6. If it does not, then the case of 'scale economies' gets omitted (see III.6 below) but nothing of the rest of our analysis is affected.

to be made in it as it stands irrespective of whether we want to analyze the problems of ZIRE or not.

III.6. The discussion so far has used the first two constraints mentioned in III.1 above. We now come to the third, i.e., the availability of finance for investment. We want to construct a relationship connecting the minimum level of profits needed to finance a particular level of investment and the corresponding rate of growth. Following Wood we shall call it the finance frontiers.

III.7. The Finance Frontier: The *finance frontier* is built up of three com ponents: the financial asset ratio, the external finance ratio and the gross retention ratio. First, let us define some relevant variables.

F = Minimum necessary acquisition of financial assets, given any level of investment I.

Financial assets are needed for providing the firm with liquidity. The liquidity is needed by firms for the usual reason of uncertainty of future, short run needs of funds and uncertainty of the availability of short run credit.

X = Maximum available amount of external finance for current investment and working capital.

R = Amount of internal finance available to the firm. In general, it is a function of its profits P.

III.8. The financial asset ratio is defined as

$$f = F/I$$

with $$F = F(I)$$

We assume that F' > 0 since the minimum acquisition of financial assets increases with the size of investment. We can assume that F' is constant for simplicity. The proportion depends on the target liquidity ratio. (It also depends on the rate of depreciation of existing assets).

In general F(O) < O. At zero investment, there is no need for positive liquidity and the firm can afford to be in net indebtedness (Fig. 3).

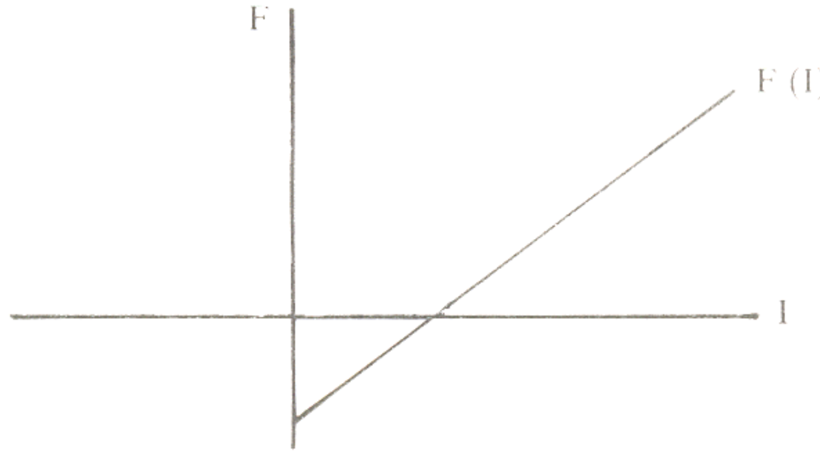

Fig. 3

F = F/I is obviously changing, but rather slowly. We lose nothing and gain in simplicity by assuming it to be a constant.

III.9. The external finance ratio

$$x = X/I$$

is the most important one for the present analysis. For the competitive capitalist economy, there is at any point of time a maximum of indebtedness a firm permits itself. As a rule very little of current investment is done by selling shares in such an economy. The gearing ratio defined as the ratio of debt to the value of total assets depends on the interest rate and the attitude towards uncertainty of the management. Hence x (I) denotes the *upper limit* of external finance per unit of investment. It also depends on the capital market's evaluation of the firm, partly reflected in the interest rate the firm has to pay. We assume that the limit to new borrowing capacity rises in proportion to investment. Once again X(O) < 0 and very small firms may not be able to afford any outstanding debt at all (Fig. 4).

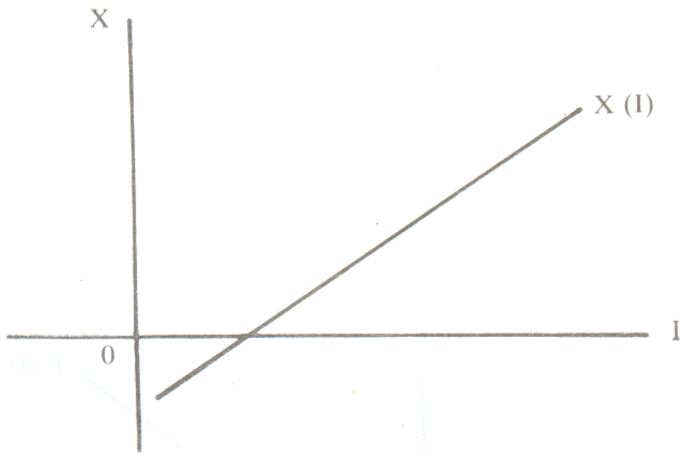

Fig. 4

III.10. The gross retention ratio is the proportion of retained earning R to profit or

$$r = R/P$$

As explained in II.8 above, this variable, capturing the dividends payments policy, is one of the major concerns of our firm.. A priori, .it is possible to argue that for a firm making a loss, all of net profits are retained while firms with positive net profits payout a constant proportion individual. If the critical level of gross profits at which the firm breaks even is denoted by P_o then we have a broken curve like in Fig. 5.

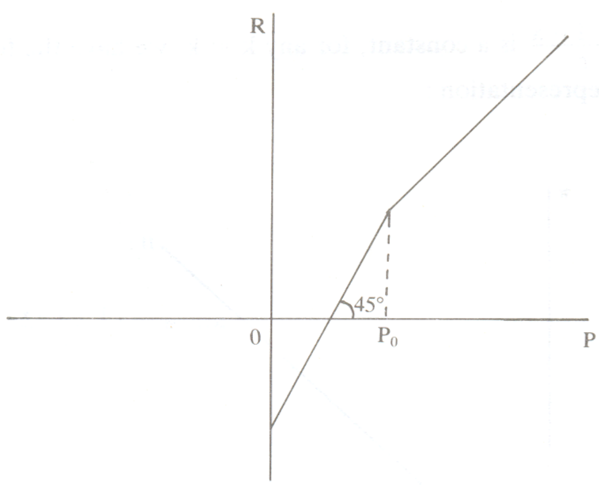

Fig. 5

In this diagram, P_o is the level of gross profits at which net profits are zero, i.e., gross profits plus non-trading income equals depreciation. Up to P_o, therefore, the firm pays neither dividends, nor taxes and R(P) is a 45^o line. Beyond P_o, a constant proportion is paid out in taxes and dividends.

III.11. Wood in his model takes f(I), x(I) and r(P) all three as constants, independent of I and P. He suggests that this is a purely simplifying assumption that leaves the qualitative properties unchanged. We shall not go into the validity of the claim since *the essential feature of ZIRE would be the non-constancy of x.* Hence we first briefly sketch his solution for purposes of later comparison.

III.12. **Equilibrium in Wood's Model**: Let a rate of investment I be given. Then the minimum necessary financial outlay is (1 + f)I of which the firm can raise xl outside. Hence retained earnings R must satisfy.

$$R \geq (1+f)I - xI \quad = \quad (1+f-x)I\,.$$

But $\quad R \quad = \quad rP. \quad$ Substituting, we get

$$P \geq \frac{(1+f-x)}{r} I\,.$$

Hence, dividing by the volume of sales, V we get

$$\pi = \frac{P}{V} \geq \frac{(1+f-x)}{r} . \frac{I}{V} \quad = \quad \frac{(1+f-x)}{r}\left(\frac{V-\hat{V}}{V}\right)\frac{I}{(V-\hat{V}}$$

$$= \quad \frac{(1+f-x)}{r} gk \qquad \text{…..(2)}$$

which is called the finance frontier, if.

Since $= \frac{(1+f-x)}{r}$ is a constant, for any k = k_1 we have the following diagrammatic representation:

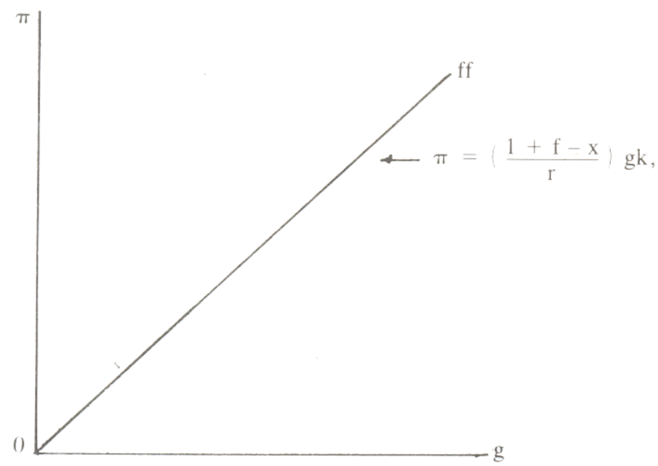

Fig. 6

Since x < 1, (1 + f + x)/r is a positive constant. Combining Figs. 1 and 6, the simplest Wood-equilibrium is given by Fig. 7.

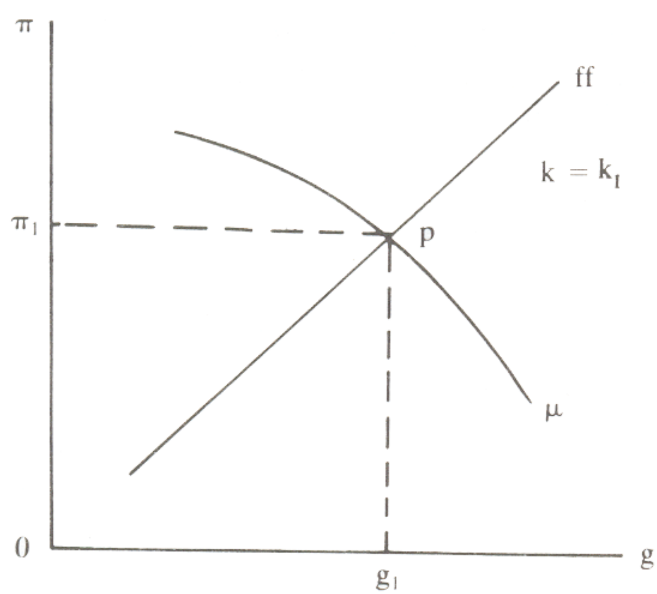

Fig. 7

The point P (π_1, g_1) yields the maximum rate of growth and the corresponding profit margin g_1 and π_1 respectively associated with a value of $k = k_1$.

III.13. In the simple model, as k rises both the curves shift upward. If g is held (artificially) constant then a rise in k swings the finance frontier (line) anticlock-wise so that π rises and g falls. If the finance frontier is held fixed and k rises then (in the simple model) μ shifts north east so that π and g both rise. In general, therefore, as k rises, π must rise but there will be opposing forces in action on g. Increased financial needs reduce g, increased demand and profitability raise it. The *outcome* depends on which force is stronger. If μ shifts out strongly then g could stay constant *or* even increase. One possibility to close the system is to postulate diminishing returns (of k on μ) so that μ shift out by progressively smaller amounts. (Fig. 8).

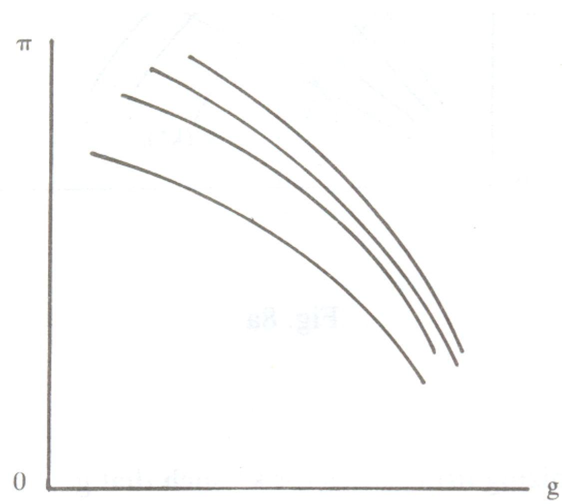

Fig. 8

The joint action then gives us a series of intersections that trace a curve concave to the π-axis (point P_i). Fig. (8a).

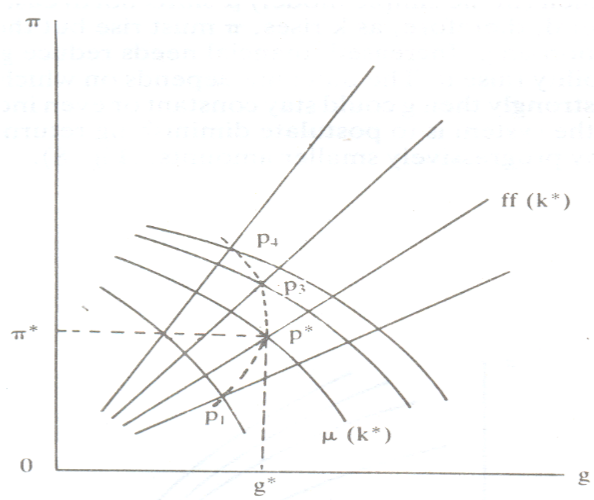

Fig. 8a

The growth maximizing firm settles for k* such that g*, the highest value of g is obtained[7]:

III.14. A serious modification of the previous model takes place if we use Fig. 2 of III.5 above rather than Fig. 1 of III.3 to depict the shifts of the curve. A milder modification is to consider intersections in the range ab of Fig. 1 itself. The second situation we can call increasing returns, the first, Marshallian economy of scale.

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7. Note that if a set of regular, quasi-concave managerial preferences over π and g are provided then the equilibrium will necessarily have a lower rate of growth and higher profit margin as compared to those in P*.

III.15. **Increasing returns / Technical progress:** Here the equilibrium has occurred in the rising part of the μ-frontier (Fig. 9).

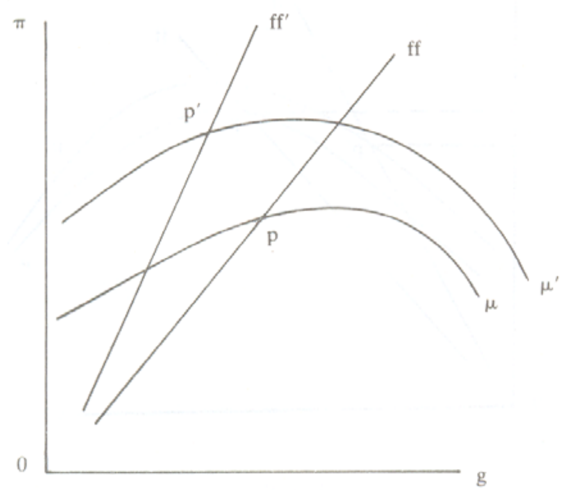

Fig. 9

Now as ff swings, both π and g fall; as μ shifts both rise. Hence the combined effect can go either way. However, considering the shift in ff alone, one gets a clear picture of the loss due to rising financial costs in the presence of increasing returns. One also gets an interior solution as bonus. If, for example, the state provided a subsidy equalling the difference between ff' and ff then higher k, and g would go together.

Also it is to be noted that in spite of the indeterminacy of P', our model does provide a way of handling increasing returns, unlike the conventional competitive model which just breaks down.

If μ shifts very slowly then we are likely to end up with a new equilibrium that gives both lower π and g (Fig. 9a).

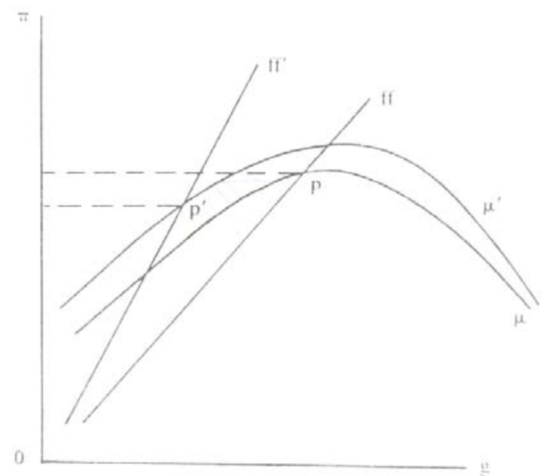

Fig. 9a

Here demand rises by much less relative to costs. Higher costs are met by cut ting down on retained earnings investment and profit margin. A higher k, however, entails a larger capacity to produce. Product price might even fall.

III.16. **Economy of Scale:** In this case the shift in π follows our fig. 2 above. Higher k initially implies a lower π due to extra costs. It always implies a higher g. The profit margin catches up only after a point (Fig. 10).

In this situation, for a while the same 1T can be generated with a larger size (k) with a lower rate of growth[8]. This is due to the larger cost associated with a higher scale dominating over the potential benefits of higher selling costs and technical efficiency.

III.17. In such a situation, the equilibrium after a switch to a higher k depends on the extent of increase in financial costs. Let μ shift to μ'. If ff shifts to ff' then the nature of solution (P') is exactly like Wood's original formulation. If interest costs are so high that the shift in ff takes it to ff'' instead, then we get our previous case of increasing returns and/or technical progress (P"). The firm is forced to operate on the rising

8. A very similar situation arises in Solow's model where two firms could have the same p.d.v. but different size and growth rate. See Solow (op.cit.).

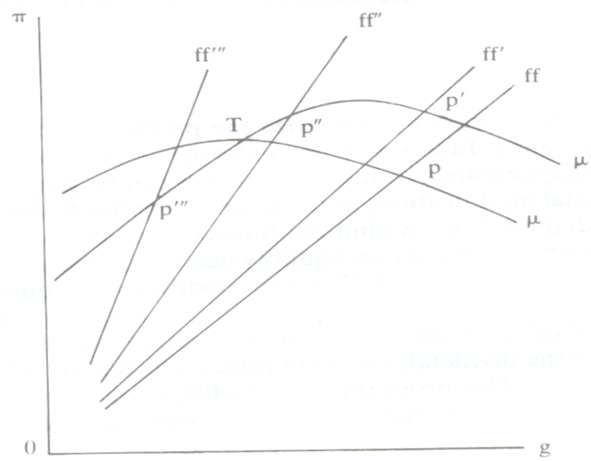

Fig. 10

part of μ’ indicating a potential social (and private) loss due to the money market. If on the other hand ff shifts as far as ff''' then the new equilibrium is at P’’’ where the firm is not only being inefficient but is doing strictly worse than before the expansion. In a non-stochastic world this is unlikely to happen so that the expansion will just not take place. In an uncertain world, however, this is not at all unlikely. A very plausible situation could be a jump in the interest rate due to government policy after the expansion has occurred. Or a fall in retained earnings r due to a lower than expected intake of revenue. Or a rise in liquidity premium f due to a fear of uncertainty of credit availability. Any of these will hit very hard precisely the thing that could be socially beneficial, viz., exploiting economies of scale.

III.18. It is of great importance to realize that the concrete difficulty is rooted in the financial constraint facing the firm. *Without this constraint the firm would simply switch to a higher scale after the point T,* a switch quite analogous to the journey down the short run average cost curves in the standard Marshallian model. Our formulation probably helps identify a gap in that theory.

IV. THE FIRM IN THE ZERO-INTEREST-RATE ECONOMY

IV.I. We can now begin to concretize the problem of the firm in a zero-interest-rate economy. In terms of the model developed above the difference made by the disappearance of the loan market can be pinpointed. It has to lead to substantial modifications of the finance frontier ff, for, the curve μ in no way depended on this[9]. Within the finance frontier, again, the financial asset ratio f depends primarily on liquidity needs which need not be affected by conditions in the loan market. The only modification required is that since in ZIRE there is no possibility of getting any loans at all (even at zero interest rate) therefore f will be higher. The retention ratio r, as we have argued earlier, depends on the dividends payment policy which is an independent variable of the model. The proportion of profits retained need not change, although in the absence of credit, the total amount paid out as dividend will certainly be higher.

IV.2. What will be violently affected is the external finance ratio x. Whereas in the capitalist economy by far the largest part of current investment comes out of undistributed profits and loans. in ZIRE the second source of funds has disappeared. In terms of our model, the only way to reach sources of funds outside the firm would be to sell shares[10]. Very little of current investment in capitalist firms is financed this way so that we have no experience to guide us in reformulating the model. What follows, therefore, is necessarily hypothetical but we shall try to make it sufficiently general to handle major possible (and plausible) alternatives.

IV. 3. Before we proceed any further, we must say a few words on exactly how, institutionally, the new economy can be expected to run. Since the bulk of company credit comes from banks and other such financial intermediaries, we can still expect this format to continue. If it seems unlikely that individual saving habits can change overnight, we need not force our firm to try to sell shares to them for every short-run investment requirements. They may but they need not. The financial intermediary can still function as a broker, buying shares in firms as they float them and paying out a return to the depositor which is now a

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9. Except for an upward parallel shift because in computing the profit margin we need no longer substract interest costs.
10. I am indebted to a referee for pointing out that this statement neglects other external sources like leasing, *Mudarabah* and so forth. A more general model would have to include such variables; our model, as we have stated several times. is but a first approximation.

function[11] of its own portfolio (rather than a constant interest rate). Just as firms often accept deposits directly from the public, they can also sell shares directly to them and in our new economy this practice can easily increase. Which mode of financing firms and individuals prefer will be situation specific and largely determined by their attitudes to risk.

IV.4. We are making no distinction between partnerships and joint-stock companies because which way the firm decides to finance its investments is determined primarily by its attitude to risk, along with the size of the investment. The latter we shall study in detail below but we shall not attempt a fully rigorous analysis of the former problem in this paper.

IV.5. **The Modified Finance Frontier:** To postulate a relationship between external finance X and investment I in a situation in which X consists not of borrowings but of shares leaves us with a wide field of choice. The simplest and most plausible one that we can think of is an extension of share financing of *initial capital* of private firms. There is ample empirical evidence that both very small and very large outlays are financed primarily from internal resources. It is in the middle range that share-financing occupies a significant proportion of total capital outlay. It is reasonable to assume that the same relationship will hold good for current investment also. Small and large investments will be done primarily out of the company's earnings. In between, there will be heavier reliance on the share market. One can see why this pattern is not unreasonable to expect. For relatively small investments (may be in working capital), the transaction costs of going to the public are usually too high; it is not worthwhile in terms of administrative costs and even the credibility of the company. For relatively large investments, usually the risk factor encourages a reliance on internal resources, undistributed profits. As an engine of growth profits are matched only by technical progress. However, we must note that debentures partially convertible into equity have found significant use for financing current investment in several countries recently. Analytically, these assets can be viewed as a combined package of equity and borrowing and hence need not deter us from continuing the analysis in terms of portfolios with 'pure' assets. Writing XS for external (share) financing we can write

$$X^S \;=\; X^S \;=\; X^S / I.$$

This modification is the heart of the matter in the present context and will cause considerable differences. It will be worth our while, therefore, to work out its implications carefully. In the process we shall also clearly underline the cases wherever simplifications of a purely technical nature are introduced.

11. Could be some sort of an average or mean return.

IV.6. Let us recall how the finance frontier was set up (III.12 above). It followed from an inequality on retained earnings:

$$R \quad = \quad r\,P \geq (1 + f)\,I - xI,$$

where r, f and x were all constants. This is now modified into

$$R \quad = \quad r\,P \geq (1 + f)\;1 - x^s\,(I).\;I$$

where x^s (I) has a specific shape.

Hence $$\pi \quad = \quad \frac{P}{V} \geq \frac{(1+f)}{r}\frac{I}{V} - \frac{X^s(I)}{r}\frac{I}{V}$$

Since $$I \quad = \quad Vgk$$

therefore, $$\pi \quad = \quad \frac{(1+f)}{r}gk - \frac{X^s(I)}{r}gk\,.$$

For a given k this results in an inequality (and the corresponding equation) in three variables, π, g and I, rather than in π and g alone. As it is, this is not a major technical problem if x^s is differentiable but it does complicate the algebra, especially because, as we shall see, the shape of x^s is complicated. In addition, it prevents us from drawing diagrams and setting up easy comparison with the parent model. Hence we simplify by assuming that x^s (I) can be replaced by $x^s(I(g)) = x^s(g)$ preserving all the qualitative properties. Now,

$$\frac{\partial x^s}{\partial g} \quad = \quad \frac{\partial x^s}{\partial I}.\frac{\partial I}{\partial g} \quad = \quad Vk\frac{\partial x^s}{\partial I}$$

where Vk is an index of the capital stock in use (as long as prices are stable). It is certainly positive; we *assume that it is roughly constant within the model* so that the curve $x^s(g)$ is a replica of $x^s(I)$. This is a purely technical simplication, made in order to focus attention on more immediate problems, and help geometric representation[12].

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12. Since $\partial x^s / \partial g = Vk\,\partial x^s / \partial I$ and Vk is constant, there is no problem of local analysis even if the assumption is dropped. But the geometry gets messy.

IV.7. The equation (2) defining the ff line is now replaced by

$$\pi \quad = \quad \frac{(1 + f - x^s(g))}{r} gk = \pi^s(g,k) \qquad \text{…… (3)}$$

with $$x^s = x^s(g), \quad x^{s^*} \le 0,$$

$$\exists\, x^{s^*} \quad \text{s.t.}$$

$$x^{s^*} \ge x^s \qquad \text{…….(4)}$$

Equation (4) incorporates the assumptions made earlier about the nature of external financing in the new environment. $x^{s^*} \le 0$ implies that share financing per unit of investment rises at a falling rate; it is a concave function. Occasionally we might even stipulate it to be a bounded function. The other inequality states that this function is single-peaked; it has a maximum value of x^{s^*}. We do not want to defend it as anything but a matter of convenience; however, its utility is demonstrated while drawing the diagram for x^s. See below IV.8 onwards.

From (3), we get

$$\pi \quad = \quad \frac{(1+f)\,k}{r}\; g \;=\; \frac{k}{r}\; x^s(g)g$$

$$= \quad \alpha\, g \;-\; \beta\, x^s(g)\, g$$

where $$\alpha \quad = \quad \frac{(1+f)\,k}{r}\; \beta \;=\; \frac{k}{r}, \frac{\alpha}{\beta} = 1 + f$$

hence $$\frac{d\pi}{dg} \quad = \quad \alpha - \beta\,[x^s + g\, x^{s'}]$$

and $$\frac{d^2\pi}{dg^2} \quad = \quad -\beta\,[x^{s'} + x^{s'} + g\, x^{s'}]$$

$$= \quad -\beta\,[x^s + g\, x^{s'}] \;=\; 0 \qquad \text{….(6)}$$

From (5), π has an extremum if

$$\alpha \quad - \quad \beta\,[x^{s} + g x^{s'}] = 0$$

or, $$x^{s} + g x^{s'} = \frac{\alpha}{\beta} = 1 + f \qquad \ldots(7)$$

Since g, x^s and f are positive, this will necessarily have a solution only if $\geq \bar{I}$ and $x^s \leq x^{s^*}$ for beyond that point $x^{s'}$ is negative[13]. Once $x^{s'}$ turns negative, the solution of (7) depends on its size. We first take up the easy case where (7) holds.

$$x^{s}\left(1 + \frac{g}{x^{s}}\frac{dx^{s}}{dg}\right) \quad = \quad 1 - f$$

Call $\frac{g}{x^{s}}\frac{dx^{s}}{dg}$ the elasticity of external finance ratio (with respect to the growth rate), δ. We have

$$x^{s}(1 + \delta) = 1 + f \qquad \ldots\ldots(8)$$

Although we are not interested in looking for a maximization of the profit margin, equation (8) yields the condition for it, should it be deemed to be necessary in any particular situation. $\frac{1}{1+f}$ is the liquid financial holdings per rupee of investment (see III.12 above). x^s is the proportion of external financing in investment. The product yields the amount of external financing per rupee of financial outlay and this is inversely related to the elasticity of x^s with respect to g. The relationship between x^s and f can be depicted in the following diagram (Fig. 11).[14]

The curve is asymptotic to the two lines $x^s = 0$ (i.e., the δ - axis(and $\delta = -1$ (the horizontal). It has a position given by the value off. For any given δ, x^s and f move together; for a given f, the lower the elasticity of

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13. I is the value of investment I that yields the maximum $x^{s'}$.
14. Fig. 11 plots different values of f associated with x^s that satisfy equation (8). But given any f, the position of the curve APB is fixed; this is the sense of the previous statement. Changing f simply shifts the curve appropriately.

supply of external finance, the higher is the size of external finance (since we are ruling out negative values of δ in the relevant region). As long as f stays put, profit maximization implies that a firm offset a rise (say) of δ by an equal percentage reduction of x^s.

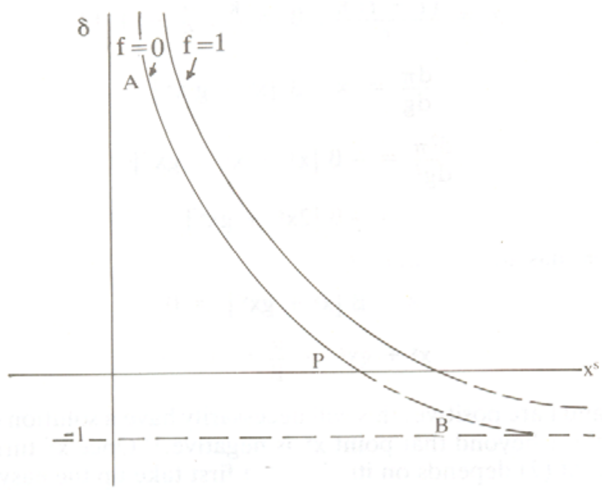

Fig. 11

One should take this result with some moderation. Even with a constant δ, introduction of revenue uncertainty will reinforce liquidity preference in the firm.

IV.8. **Construction of the General Case:** Let us now get out of the ‘easy’ case and see if we can define the x^s function - and thus, the new ff curve - more clearly and generally so as to set up a precise comparison between the earlier finance frontier and the new one. As yet we are lacking a precise basis of comparison since all that we have postulated is that a proportional external finance ratio has been replaced by a variable-proportion equity/investment ratio.

To this end, let us go back to the gross equity and its stipulated relationship to investment, I[15]. Diagrammatically, this is represented by Fig. (12).

15. See IV.5 above.

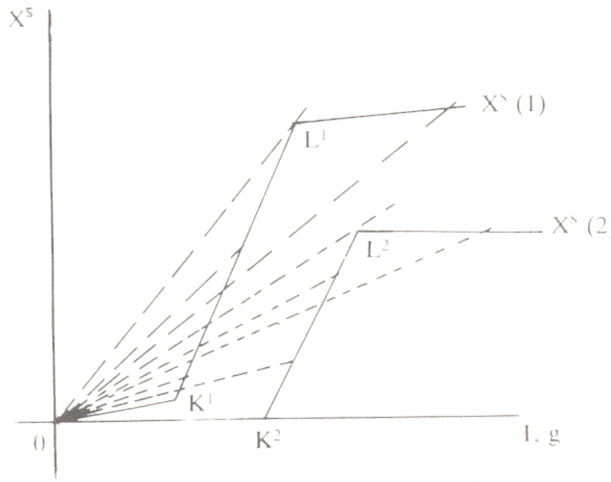

Fig. 12

We have drawn two possible shapes, $X^S(I)$ and $X^S(2)$, the second one being the more extreme case. Along $x^s(1)$, the ratio $x^s = X^s/I$ is never zero, though it is 'small' to the left of K^1 and to the right of L^1. Along $X^S(2)$, x^s is zero over $0K^2$, rises till L^2 and falls thereafter.

In either case, x^s is non-decreasing till a turning point is reached; along $X^S(1)$ it is always increasing till L^1 while along $X^S(2)$ it is zero till K^2, and increasing till L^2. Beyond a certain point, however (L^1, L^2), x^2 falls but it never reaches zero unless X^S turns down and hits the horizontal axis again, something that we assumed away. Thus, generally, x^s will rise between zero and some positive value of I**,** reach a peak (say at I) and asymptotically go to zero thereafter. (See Fig. 13).

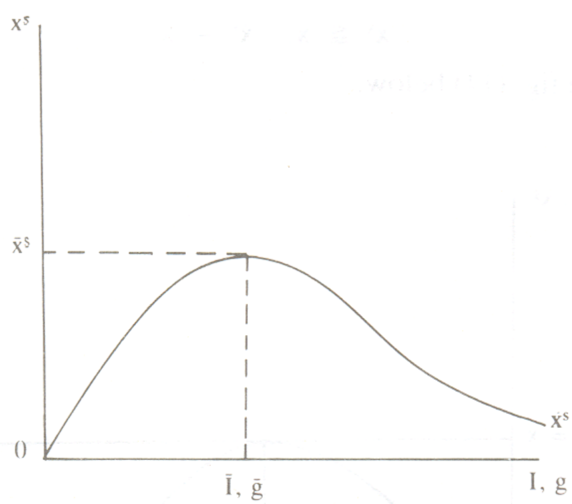

Fig. 13

Given this general shape, in order to compare the new situation meaningfully with the old one two pairs of cross-correlations are necessary. First, how does $\overline{x}^s$ compare with the previous constant x(= X/I)? And second, where does $\overline{g}$ lie *vis a vis* the previous equilibrium value g* as derived by the intersection of the μ and ff curves of figure 7?[16] Unfortunately, both are comparisons of size and crucially depend on empirical information. So, once again, we shall resort to the familiar procedure of (a) deriving all the major possibilities (b) make a judgment about the more probable cases and (c) most important of all, now that the system has a degree of freedom, study the essentials of policy measures.

IV.9. To perform the aforementioned exercise efficiently, note that nine possibilities in all are involved; they involve combining the relations

$$\overline{x}^s = x^s_{max} \gtreqless x \quad \text{and} \quad \overline{g} \gtreqless g^*$$

To do this with some economy of effort. take first the case

$$\overline{g}^s = x \quad \text{and} \quad \overline{g} = g^* \qquad \text{.......(9)}$$

In words, the relations (9) imply that the maximum ratio of equity-financing to investment is the same as the previous ratio of loan-financing to investment and that this maximum occurs at a value of $g\,(=\overline{g})$ which is the previous equilibrium value of g (= g*). It may not be obvious but these two specifications help fix the position of the new finance frontier completely relative to the old one.

Let us demonstrate this in steps. The first is the comparison of x' and x. Since x was a constant and $\overline{x}^s$ the maximum of a single-peaked function, they are related in the present case by the inequality

16. An unfortunate problem of symbols: in figure 7, the equilibrium was denoted by (π_1, g_1) and not (π^*, g^*) because it was indexed on k = k_1. The g* value used here is the same as g_1 - it simply indicates the

original Wood - equilibrium.

This is shown in Fig. (14) below.

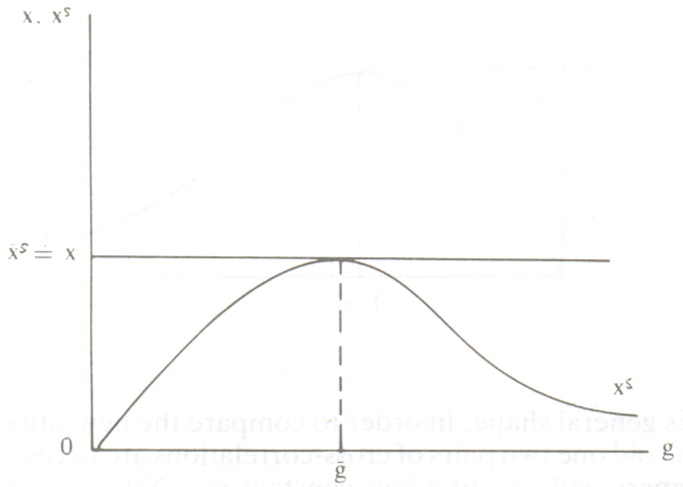

Fig. 14

Secondly, let us compare the old finance frontier ff and the new one. Recall the equation of ff:

$$\pi \quad = \quad \frac{(1+f)}{r}\,k\,g - x\,\frac{k\,g}{r} \quad = \quad (a-b)\,g,\ a > b,\ b = \frac{kx}{r} \qquad \ldots.(10)$$

The new finance frontier will be given by a relation

$$\pi^s \quad = \quad (a - b^s(g))\ g, \quad b^s(g) \quad = \quad \frac{k}{r}\,x^s\,(g) \qquad \ldots.(11)$$

Clearly, $\pi^s \quad = \quad 0$ when $g \quad = \quad 0.$

To place ff^s relative to ff, we have to find out, for a given g, what is the value of π^s as compared to π? To answer this, observe that

$$\pi \ - \ \pi^s \ = \ (b^s \ - \ b)\ g \qquad \ldots.(12)$$

But $\qquad b^s - b \ = \ \frac{k}{r}\,(x^s\,(g) - x) \ \leq \ 0$ by (10)

Thus, $\pi \leq \pi^s$ generally, and $\pi = \pi^s$ when g = 0 and again when $x^s(g) = x$, i.e., when $x^s(g) = \overline{x}^s \;\; \mathrm{I} = \overline{\mathrm{I}}$

Thus, ff^s starts at the origin, lies above ff until $\overline{\mathrm{I}}$, (which is equal to 1^* in the present case) where π^s equals π and subsequently, again lies above ff (Fig. 15).

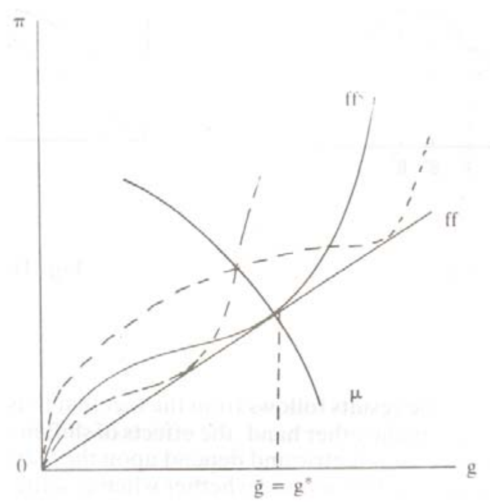

Fig. 15

By inspection of diagram, it is obvious that any shift in μ that leaves ff^s unaffected will raise π and lower g as compared to ff[17]. The reason is not far to seek. Share-financing has, the previous loan financing as its upper bound, making a higher retained earnings and hence a higher π necessary for the same rate of growth. In some sense, external financing is now more stringent than it previously was.

It is also obvious that if the maximum of x^s is achieved at $\overline{g} \neq g^*$ then equilibrium occurs at lower g and higher π no matter if $\overline{g} < g^*$ (Fig. 16a) or if $\overline{g} > g^*$ (Fig. 16b).

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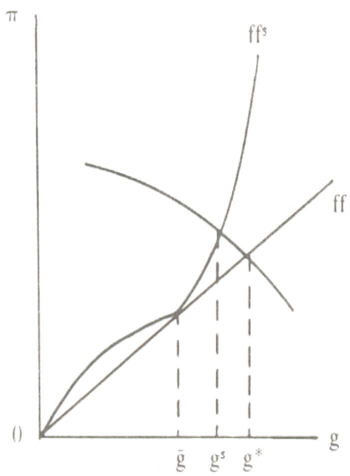
17. Geometrically, this is because ff^s lies to the north west of ff; If μ shift, its intersection with ff^s will always lie to the north west of its intersection with ff. The two dotted lines for ff^s are alternate positions when $\overline{g} \neq g^*$.



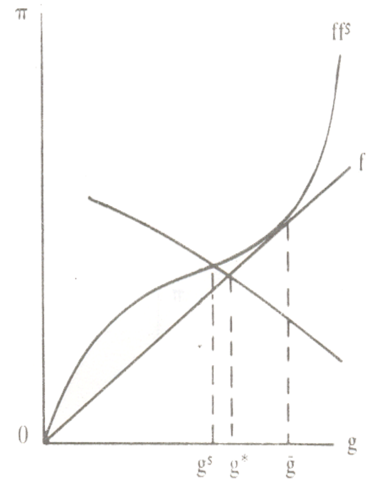


Fig. 16a Fig. 16b

The uniformity of the results follows from the fact that ff^s is rising both to the left and right of $\bar{g}$. On the other hand, the effects of shift in demand, say shifting μ outwards, are asymmetric and depend upon the value of g in question. The real distinction evolves around whether when μ shifted, the equilibrium $g >^{<} g$, i.e. whether we are in the equity expanding regime (to the left of $\bar{g}$) or in the equity contracting regime (to the right of g). Demand expansion (outward shift of μ) in the equity expanding regime brings the equilibrium in ZIRE closer to the Wood-equilibrium for equity is catching upon the previous credit availability; demand expansion in the equity contracting regime makes higher and higher profit margins and retained earnings necessary to support a given rise in the growth rate, g. The ZIRE equilibrium diverges from the Wood-equilibrium.

IV.10. We can quickly dispose of the case where

$$\max x^s < x,\ \bar{g} \underset{<}{\overset{>}{-}} g^* \qquad \ldots(13)$$

In this case, the conditions of external financing are worse than in the previous one because the highest ratio of equity to investment ($\bar{x}^s$) falls short of the previous ratio of loan financing to investment (x). The analogue of fig. 14 is now given in Fig. 17.

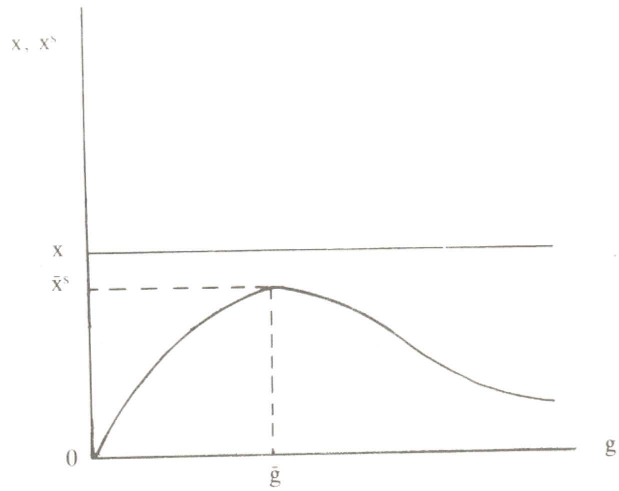


Fig. 17

The pressure on retained earnings rises further so that a higher π will be needed for sustaining every g. The new equilibrium can be depicted as in Fig. 18.

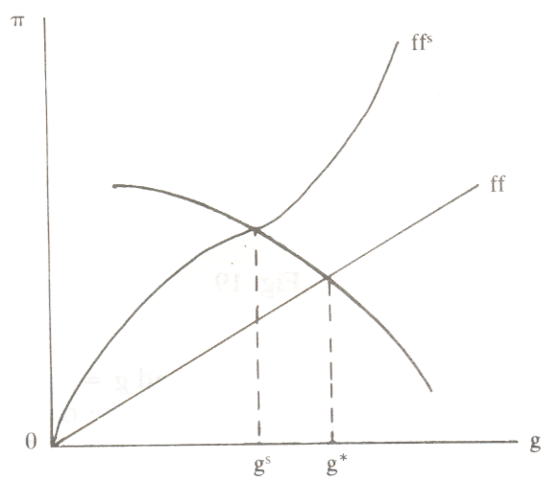


Fig. 18

Thus, for $\overline{x}^s < x$

the new equilibrium $\pi > \pi^*$ and

the new equilibrium $g^s < g^*$.

IV.11. **The ZIRE Equilibrium:** We come now to the most interesting case where

$$\max\ x^s = \overline{x}^s > x\ . \qquad \ldots(14)$$

We call it most interesting not only because it yields a richer basket of possibilities but also, as we shall argue, on a priori grounds this seems to be the more probable case since $\overline{x}^s$ might be a parameter more directly affected by policy measures than would seem to be the case with x. (See item IV.12 below) .

The relevant comparison of x and x^s is now depicted by Fig. 19. From the relation

$$\pi - \pi^s = \frac{k}{r}(x^s(g) - x)\ g$$

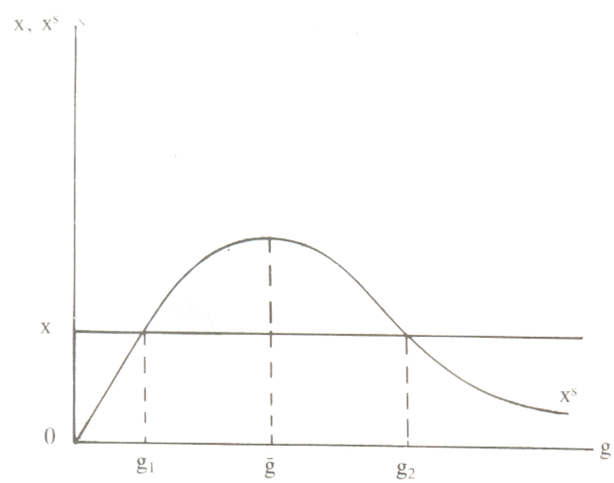


Fig. 19

we can see that π equals π^s at $g = 0$, $g = g_1$ and $g = g_2$ of fig. 19. Thus he new ff^s curve will cut the old ff twice (excluding the origin) and we get the overall picture as in Fig. 20. Because x^s is supposed to be single-peaked, $(\overline{g}, \overline{x}^s)$ indicates, as before, the maximum point of this function. This diagram should be studied carefully. It provides the situation where equity-financing exceeds loans for certain values of g - in the range (g_1, g_2) while falling short of the latter for either $g < g_1$ or $g > g_2$.

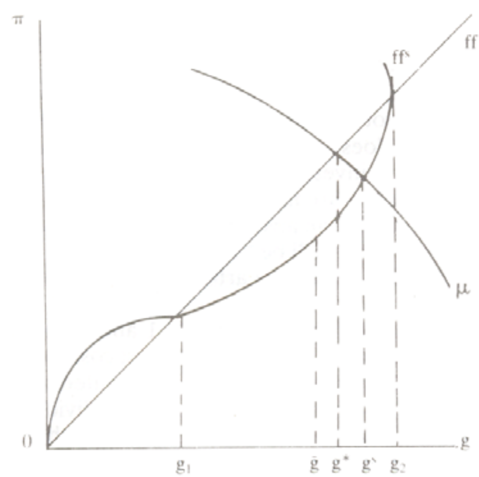


Fig. 20

If the curve μ cuts ff^s either to the left of g_1 or to the right of g_2, we get the two previous cases. If, however, the condition

$$g_1 < g^* < g_2 \qquad \text{......(15)}$$

holds then

equilibrium $g^s > g^*$ and

equilibrium $\pi^s < \pi^*$

irrespective of whether $\overline{g}$ (of fig. 19) $\substack{> \\ = \\ <}$ g^*.

IV.12. It should also be observed that what matters is the spread between g_1 and g_2 and not the amount by which $\overline{x}^s$ exceeds x, as long as $\overline{x}^s > x$ holds. That is, for a higher growth rate to obtain *what matters is whether* *equity financing exceeds the previous loan financing over a wider range of investment projects rather than whether it exceeds loan financing by a massive amount for a small band of investments alone.*

It is in this context that the claim was made above that the present case merits more attention. For, as a moments' reflection suggests, what the government can, and probably is going to regulate in the present system is x^s directly just as it regulated, the interest rate in the previous regime. Further, as we noted earlier in the context of institutions, the present system is going to operate like a Unit Trust generally where public institutions collect individual savings and invest them so that the equity investment ratio will come under direct policy intervention. This is the new degree of freedom in the system and what makes it richer than the earlier case. In what follows, therefore, we shall assume that condition (15) always holds so that compared with a capitalist firm, the firm in ZIRE will have a higher rate of growth and a lower profit margin.

It is hard to either accept or reject this condition on logical grounds alone. If it fails, of course, we are back to the two previous cases; nothing is lost analytically but a wide range of outcomes gets ruled away. But on a priori grounds it seems hard not to expect an Islamic state to actively ensure that the unavailability of credit does not become a severe hindrance to the growth and profitability of firms and investors. Since most Muslim authorities agree that many speculative contracts are not permitted but the public authorities have the duty, and of course the means, to rationalise the share market, we can safely assume that the latter will be "well behaved", meaning, in the present context, that investors will not face arbitrary, exogenous constraints on the supply of funds.

This result neglects the footnote 9 to IV.1 above that the curve μ itself might shift upwards due to the absence of interest cost (payments) in calculating the profit margin. However, net profits, computed *after* the deduction of dividends will have to accommodate a larger dividend deduction. On balance, μ^s is unlikely to go out too far. To the extent that it does (curve $\mu^{s'}$), the growth rate will be further enhanced and the fall in the profit margin, as compared to the capitalist economy, partially modified.

IV.13. Under our assumptions, thus, the conversion of credit into equity capital seems to be generally beneficial to the firm's growth prospects, especially for investments of the intermediate range. As compared to Wood's model, the proportionality relationship of external finance with investment is replaced by a non-linear one, induced by the evidence on the behaviour of equity capital. At the same rate of growth the firm in

ZIRE makes do with a lower profit margin. In some sense there is a greater efficiency in the use of investible resources.

This throws an interesting light on the net effect of the credit-activity of modern financial institutions on the economy. They help the system maintain a higher rate of profit but a lower rate of growth as compared to a Shares' economy.

IV.14. We can briefly do some of the comparative static analysis analogous to those of Ch. III, Sections 13, 14 and 16 above. First, as k rises, the curve μ^s shifts above in the same way as before. It may have an upper bound. The new ff curve in which k enters multiplicatively will swing up and around anticlockwise. Hence the general pattern of III. 13 will hold. However, the firm in ZIRE has a better chance of handling either economies of scale or increasing returns/technical progress kind of phenomena. For example, referring to Fig. 10 (III. 16) and replacing ff by ff^s we have the following situation (Fig. 21). Precisely because ff^s first dips and then rises the chances of an intersection of ff^s with μ^s *above* the intersection of $ff^{s'}$ with $\mu^{s'}$ are very much lower. Why? As we know, a higher scale is less profitable at low levels of output. But in ZIRE, the profit margin required by a firm to finance a certain scale expansion *is less than in a capitalist firm and it falls for a while.* Even in capitalist economies, for that matter, it is large firms with less of credit financing that are better placed to handle scale economies. In ZIRE this becomes almost automatic.

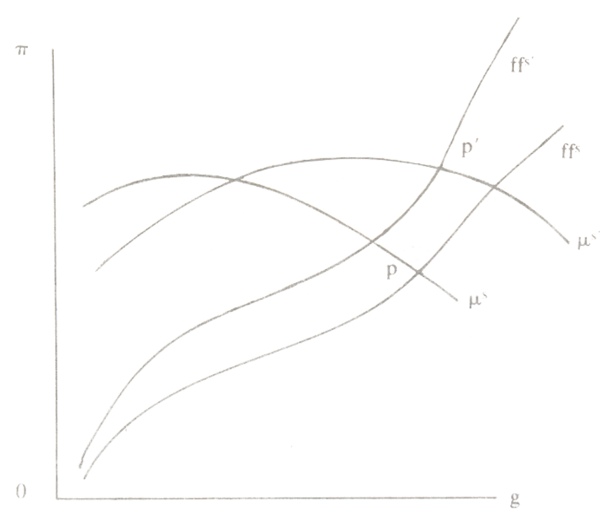


Fig. 21

In the equity-expanding phase, unhindered by large interest payments on sizable capital, the ZIRE firm seems to be in a much better position to weather the 'uneconomic' phase of a project with sizable scale economies. Our analysis perhaps provides the concrete role played by the financial constraint in this process.

V. LIMITS TO COMPETITION IN *ZIRE*

V.I. We now take up a question of some academic importance not so far posed either in Wood's model or in our reformulation. It is concerned with the degree of competition. Specifically, we want to explore the consequences of an increasing degree of competition in both the models.

Admittedly the structure laid out does not directly accommodate the pure, profit-maximizing, competitive firm. In fact Wood's work is an explicit repudiation of such a system. However, it is not an unimportant question to ask, what differentiates a highly concentrated from a numerous industry? We shall first show how to answer the question in a capitalist economy and then turn around to explore the consequences of varying degrees of competition in ZIRE.

To work out the consequences of increasing degree of competition we shall have to see how to modify the J.L-ff diagram for this situation. To repeat our previous remark, there can be no pure competition in our model; the conventional static, zero-profit equilibrium corresponds to the *origin* in the 'g-π' space. That is why we have been posing a relative question (what happens with *increasing* competition?) rather than an absolute one (what is *the* competitive solution?).

Increasing Competition: The Opportunity Frontier: First, the opportunity frontier. **In** any given industry if the number of active firms increases, the μ-curve will both shift towards the origin and become inelastic. It shifts towards the origin because at any given rate of growth a more concentrated industry shows a higher profit margin than a more competitive one. It becomes more inelastic because the higher the competition the greater the sales effort needed to maintain a given rate of growth and hence the greater is the reduction in the profit margin. **In** effect, to re-do our earlier diagram we are in a four-variable world; each μ-curve is now specific to *two parameters,* a given k and a certain number of rival firms, n. (Fig. 22).

Given a certain $k = k_1$, as the number of competing firm n increases (from n_1 to n_2) the opportunity frontier shrinks towards the origin and pivots towards the g-axis from μ^1 to μ^2. Comparison of μ_1 and μ_2 should be carefully made. μ_2 states, for example, what and where would be the growth-profit trade-off if the capital coefficient stayed unchanged but the firm simply faced a number of rivals n_2 that is greater than was along μ_1 i.e. n_1, With unchanged financial conditions, the equilibrium will shift to P^2 indicating a lower profit margin and a lower rate of growth. But as we shall see in a minute, financial conditions will *not* remain the same. (See V.2 below).

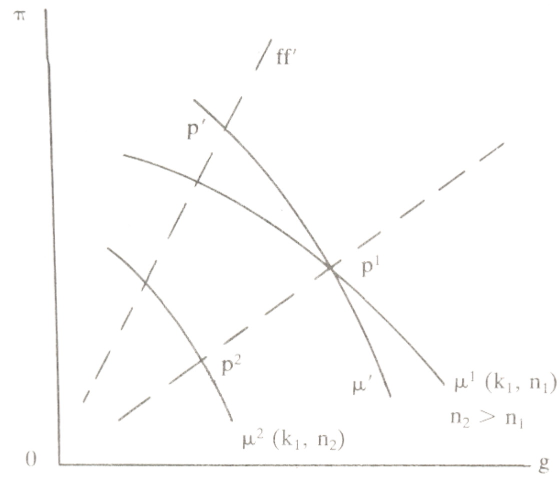


Fig. 22

We might note in passing that the firm cannot regain the original equilibrium P^1 by increasing k because a higher capital intensity raises the costs of financing growth. If it increased capital intensity so as to shift μ^2 to μ' (passing through PI) and even if a larger n did not affect ff (an untenable assumption), a higher k as we know swings ff left (to ff', say). *Thus an increased competition cannot be fully offset by increasing capital intensity* i.e., size of operations; the new equilibrium is at P' and not at P^1 any more.

V.2. **Increasing Competition: The Finance Frontier:** But as we mentioned earlier, a rise in n does *not* leave the finance frontier unaffected. To figure out how the latter changes, it is instructive to consider the financial activity of a competitive firm. For such a firm, f is zero (it holds no liquid assets for it foresees no liquidity problems) and r is one, all profits are retained. Thus,

$$P = (1 - x)\, I,$$

or,

$$\pi = (1 - x)\, gk.$$

If, further, we make the steady-state assumption then x is zero i.e., *only profits and all profits finance growth* and we get

$$\pi = gk$$

which is the von Neumann solution.

The argument that is relevant, however, is that with f = 0, π falls as x rises. Hence, we have to decide, how does x behave as n increases? The long-run steady states argument does not carryover to the problem of increasing competition directly because in the von Neumann equilibrium firms can as if work as their own bankers. One has to recall that π refers to profit *margin* and the Fisherian theory of investment in the presence of full competition makes this margin zero. All investment is done by borrowing in a perfect capital market so that no profit retention is necessary. In other words, x rises, external finance increases eventually to account for all investment, and profit *in our sense* which is a pure profit, is driven to zero. (As we shall see later, the nature of the solution for a firm in ZIRE will differ sharply on this count precisely because the nature of the loan market changes).

In general, then, as the degree of competition (n) rises and firms become relatively less significant, we can expect f to fall and r to rise (towards unity). But the equation of ff is given by $\pi \; \frac{(1+f-x)}{r}.gk$ (See III.1 above). With a falling f and rising r, the slope *falls.* Thus as n rises, the ff curve, still a straight line, *swings down towards the g-axis.* (Fig. 23)

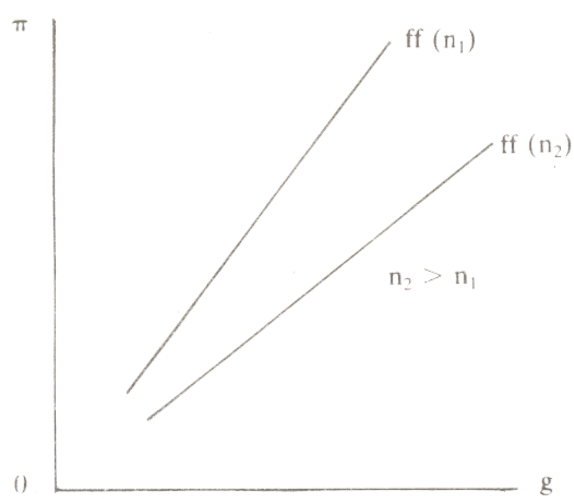


Fig. 23

V.3. **Change in Equilibrium as the Degree or Competition Changes:** The complete picture can be assembled now by putting together figures 22 and 23 (Fig. 24)

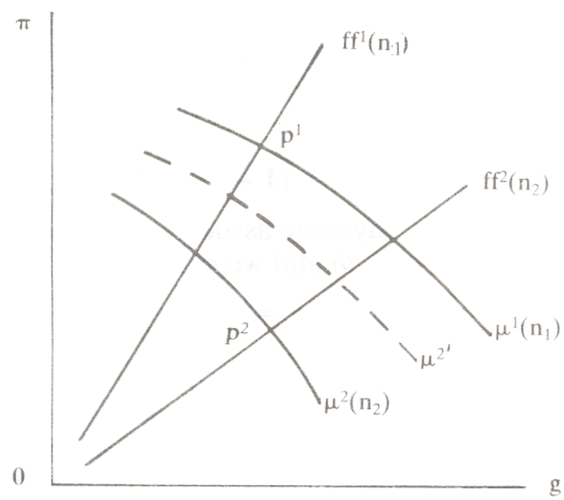


Fig. 24

The new equilibrium at P^2 will definitely be associated with a lower profit margin. If, however, as competition increases the opportunity frontier does *not* shrink very rapidly then the rate of growth need not necessarily *fall*. We have drawn a curve $\mu^{2'}$ to indicate such a contingency. Since the μ-curve captures both the elasticity of aggregate demand and its exogenous growth (say due to population growth) we can turn around this result to get the following conclusion.

Generally with a rise in n, π and g will fall; with a fall in n they will both rise. However, if demand is inelastic and exogenously growing sufficiently rapidly, then even though n rises (lowering the share of each firm), the market is growing fast enough to offset some of this reduction. Thus, π falls, but by less than otherwise and g may actually rise. Similarly, if n falls in such a situation, π will definitely rise and g may even fall. Here is a market with heavy, inelastic demand getting concentrated into the hands of fewer sellers. Outcome: a higher profit margin and a lower rate of growth of output. Markets for essentials, like food grain in India, come to mind. (Notice, we are not considering state-interventions, subsidies etc.).

But we do not want to put too much weight on the assumption of exogenous demand-growth. For one thing, we do not know about it; for another, more importantly, we do not want to clutter up our analysis of variation of number of rivals (n) with other parameter changes.

VA. **Increasing Competition and the ZIRE Firm:** Let us now try to do the second set of necessary modification in order to apply these results to ZIRE. By now it should be reasonably straightforward but some unexpected situations do crop up.

It seems that on a priori grounds the modification to the μ-frontier due to a rise in n would be no different in ZIRE as compared to a capitalist economy since they are all a function of the internal structure of the industry. The frontier will shrink and become less elastic. But the ff curve was set up on entirely different principles of financing. It was argued (IV.5-IV. 7 above) that share financing is significant only in the middle range of investment, that for very small and very large amounts, undistributed profits (and credit, in a capitalist society) bear the brunt of investment. As n rises, however, each firm is becoming a smaller fraction of the market and hence, *compared to the situa tion before the change,* there will be less of share financing and more of profit financing. Our fig. 12 now is redone to look like Fig. 25 below:-

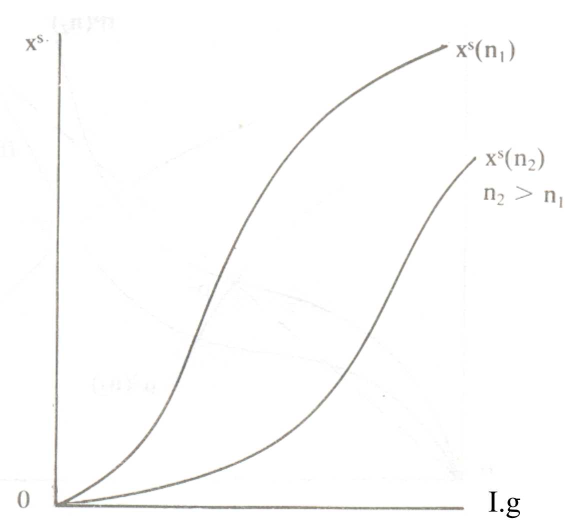


Fig. 25

At each **I,** share financing constitutes a smaller proportion of I when n is larger. $X^S/I = x^s$ is thus uniformly smaller (Fig. 26). *It will necessarily peak at a higher* value of I and g.

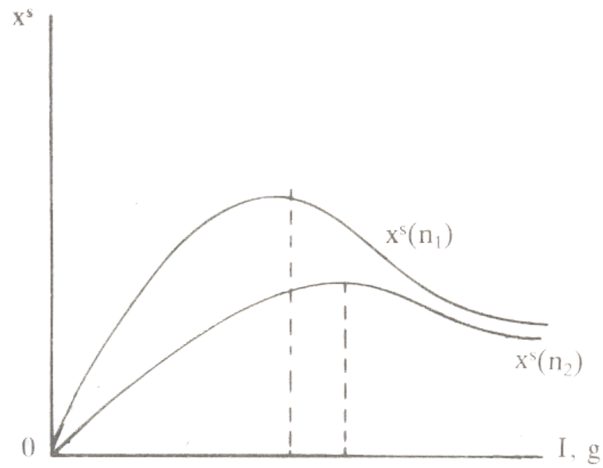


Fig. 26

I t is useful to remember that the asymmetry of the behaviour of x^s and of x (as n rises) is due to our basic postulate about the nature of the X^S function as I changes. X^S is low for low levels of investment; here, in the present case, as n rises the amount of investment that anyone firm can undertake will get smaller and smaller.

V.5. In qualitative terms it is not difficult to modify our previous analysis to accommodate this change. The new external finance curve x^s (for a larger number of rivals, n_2) will lie below the earlier one and will also be flatter (since it peaks at a higher g). As long as condition (15) continues to hold, it will simply mean that $ff^s(n_2)$ lies *above* $ff^s(n_1)$ and the points of intersection of $ff^s(n_2)$ with the previous ff will be closer together than those $ff^s(n_1)$ with ff (Fig. 27).

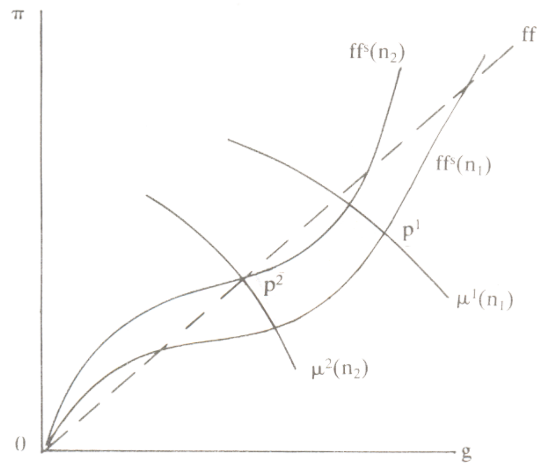


Fig. 27

In this diagram, the new equilibrium at a higher n is P^2, the original one at $n = n_l$ is at P^l. P^2 will lie necessarily to the left of P^1; it is generally likely to lie below the latter too. Thus, the growth rate will definitely be lower while the rate of profit is likely to be so. In the capitalist system, as n rises, the profit rate would definitely fall while the growth rate is likely to do so. In the regular cases, in both systems g and π would simultaneously fall as n rises. The unusual case occurs in ZIRE when at the same time as the supply of external finance (per firm) declines the investment needs for expansion stay so high that a comparatively higher profit margin is required.

V.6. **Approach to Competition:** What happens now if n indefinitely rises? The curve μ is shifting inwards and getting less and less elastic while the curve ff^s is swinging anticlockwise. In the capitalist system too μ behaves in similar fashion but ff swings *down* clockwise. In the limit both the systems go to a zero-profit, no growth state but they do so in different ways. The ZIRE firm will tend to conserve profits much longer in the earlier stages while the capitalist firm will tend to nurture growth. The shortage of external finance compels a ZIRE firm to protect its profit margin, the capitalist firm, presumably operating in a perfect capital market, has no such worries[18]. The perfect capital market is not supposed to discriminate between large or small investors.

18. The idea is not that a ZIRE firm *will* face shortage of capital. Our argument is that *everything else remaining the same,* a rise in n necessarily reduces the per-firm availability of funds.

If this result seems to be at odds with our earlier analysis of the ZIRE firm and its comparison with a capitalist firm, we would like to suggest that it simply highlights the strength of the assumption of a perfect capital market. The ZIRE model might easily represent or simulate what really happens in the real world.

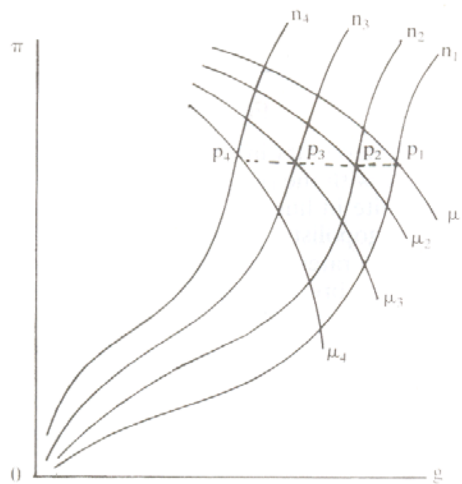


Fig. 27a

V.7. In conclusion we might briefly mention the *observable consequences* of our theory. Suppose for the sake of argument that there is slow, secular growth of the number of firms in some industry in ZIRE but the environment does not change otherwise. The industry adjusts according to its own inner mechanism hopefully as described above, but from the outside all that we get to know are the equilibrium values of (π, g_i,) as n takes on values n_i, (i = 1, 2,...). What shall we observe?

Obviously, the observation will consist of the points P_1 as in diagram 27, but it is possible that these observations, in turn, have a pattern quite unexplained by anything in the model so far. Take for example Fig. 27a for a possible configuration:

If the points P_i are plotted with a slight adjustment of scale for convenience, one gets Fig. 27b:

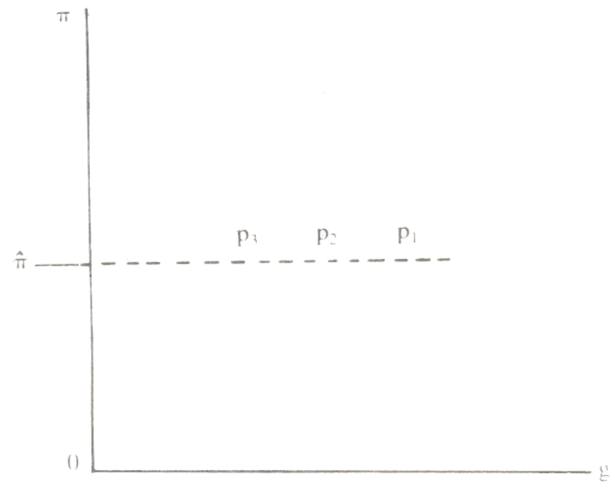


Fig. 27b

It would seem that with new entry in the industry, there is being a steady sacrifice of the growth rate with the profit margin being maintained at some π! It would, in fact, be quite in line for an observer to posit that ZIRE firms behave as Baumol's oligopolists, protecting a certain minimum profit and choosing different growth rates to adjust to competition. He would be quite wrong, of course, but we brought out this case to emphasize the need to clearly distinguish causal from descriptive models.

It is interesting to observe that a similar exercise performed on Wood's model throws up, as a *likely* solution, the situation where the points P_i are vertically arranged at some $\hat{g}$, from which, again, a conclusion would emerge that profits are a very volatile quantity. Again, seemingly plausible but not really 'explained' by the model.

VI. STRATEGIC BEHAVIOUR AND UNCERTAINTY

VI.1 This monograph is not the place for a full discussion of uncertainty in the context of large, non-competitive industries, research on which is at pre sent very active. To develop a formal model would require much more of mathematical arguments than has been used in this text; instead, we shall try to review the major *kinds* of research now available and correlate them to the structure constructed. It is of some importance to carefully sort out ideas and assumptions; otherwise the chances of getting involved in false theorizing rise rather rapidly.

VI.2. We used the pharse 'kinds of research' on purpose to highlight the distinction between two major approaches to modelling uncertainty in the theory of the firm. The first, more orthodox one, is concerned with introducing it while leaving the description of the rest of the economic environment and the behavioural rules of the game unchanged. In particular, the focus of emphasis is on the decision-making of individual agents who confront uncertainty about the future values of some endogenous variables like prices, rates of return etc., but his uncertainty does not affect the price-taking behaviour of the system. The maximization of expected utilities, incomes or profits then proceeds under suitable assumption about individual behaviour in such situ ations. In other words the description of the market and its rules are not altered - what changes is the individual agent's perception of the *outcomes* of certain actions. The agent can certainly make contingent plans and the model can be modified and/or enlarged to accommodate alternative actions that will be undertaken depending on a particular realization of the uncertain variable (s). But the critical point is that the agent still does not alter the set of variables that he can, or can try to, control as compared to a non-stochastic world. To give a concrete example, if the product price is uncertain, then the traditional treatment will seek to work out the equilibrium of *expected* profit maximization while the firm is supposed to continue to adjust output to prices.

VI.3. The alternative to this can be called strategic behaviour where the firms behave not as simply price-taking agents but also as the regulators of a larger class of control variables. It is *not* the case that strategic behaviour necessarily requires an uncertain environment but simply that it makes better sense in one. The original Cournot model of duopoly was a case of strategic behaviour described without any explicit reference to uncertainty. But one should recognize that what Cournot's assumption about reaction-functions actually did was to lock-in or resolve an intrinsically uncertain variable (i.e., the behaviour of a competing duopolist) into a predictable, non-stochastic relationship. A faithful description would be that the Cournot solution is a Nash equilibrium where a pair of dominating strategies can be shown to exist (one for each player) even though in principle each commands a set of strategies and it is a game of chance. In terms of our earlier example, a firm facing uncertain future prices is now permitted to try to enhance sales through advertisement, promotional campaigns, Product-differentiation and so forth.

VI.4. Clearly, for the problem at hand, i.e., the behaviour of a firm in ZIRE, the second type of formulation seems to be the more natural one, for, all along we argued that the role of the financial constraint is not well described in a Walrasian model of price-taking agents. But a somewhat subtler problem remains. In tJ.te present context the problem is not simply one of introducing strategic behaviour about inter-firm rivalry in the product market although that seems to be the most fruitful line to take. The problem, fully stated, is about the difference made, *to strategic behaviour,* by the fact that firms do not enjoy the right to borrow money at a positive interest rate. In other words, we first need a comparison model that describes the outcome of strategic behaviour and that *includes the role played by the financial constraint in it.*

Unfortunately, to the best of our knowledge, no such models have been worked out yet. In Wood's model the entire problem has been subsumed in the μ-frontier. A very liberal interpretation, therefore, would suggest that it is already there. This may be true but for our purposes it is too general. What is needed is a guide to the kinds of modification made of the μ-frontier by different types of strategic situation. It is hard to expect any general results. But it is only after this is done that the consequences of randomizing the finance frontier, say by making x^s stochastic, can be fruitfully investigated. An example might again elucidate the point somewhat. A problem of strategic behaviour that has been recently investigated *without* a financial constraint is the following: Suppose a firm (or a country) has monopoly over a natural resource. A rival firm (or country) is contemplating committing money to R & D, so as to find a substitute (resource or technology). What will be the optimal pricing policy of the first firm? Should it also try to engage in R & D and pre-empt the threat? Should it plan to conserve some of its stock even after the substitute is expected to come on stream? And so on (see Hoel (6) Das gupta (4), and Dasgupta and Heal (4a)). As can be expected, the answers are not easy to come by and a modification by introducing financial considerations would be far too technical for the present monograph.

VI.5. **The Cournot Model with a Financial Constraint:** In view of this difficulty we shall not pursue the pure uncertainty problem any further but try to sketch some simple illustrative examples of strategic behaviour that can give some insight into the problem in an easily identifiable format. What we do is impose the financial constraint on a model of Cournot duopoly where the strategic behaviour is subsumed in the well-known reaction function. We then contrast this with a similar exercise done with the model due to Solow (op.cit.).

Recall that in a pure Cournot model of duopoly, two firms produce a homogeneous output under zero material cost of production (which is purely a simplifying assumption that does not alter the essential features of the solution) and the market price depends on the total output of the two firms. Each firm assumes, while choosing its output, that the other party will not react but continue to produce as before, i.e. each treats the other's production as a given parameter. Letting

q_i denote the output of the i^{th} firm ($i = 1,2$)

$$q = q_1 + q_2$$

and p to be the market price,

the demand function is given by

$$p = F(q), \; F' < 0.$$

Zero costs imply that firms maximize

$$R_i = pq_i, \quad i = 1, 2.$$

The behaviour assumption implies

$$\delta q_i \; / \; \delta q_i = 0, 1, 2.$$

Let F be linear, i.e.,

$$p = a - bq.$$

Then $\qquad R_1 – q_1 (a - b(q_1 + q_2)) – aq_1 – bq^2_1 – bq_1q_2$

and $\qquad R_2 – q_2 (a - b(q_1 + q_2)) – aq_2 – bq_1q_2 – bq_1q^2_2$

Maximizing R_i over q_i yields two first-order conditions

$$\frac{\delta R_1}{\delta q_1} = a – 2bq_1 – bq_2 = 0$$

and $$\frac{\delta R_2}{\delta q_2} = a - bq_1 - 2bq_2 = 0$$

Solving for equilibrium outputs we easily get

$$q_1 = q_2 = \frac{a}{3b}. \text{ Hence } q = \frac{2a}{3b}$$

Substituting back in R_i we get

$$R_i = \frac{a}{3b}\left(a - b\,\frac{2a}{3b}\right) = \frac{a^2}{9b},\ i = 1,2.$$

Since there are no costs, in terms of our earlier relation R_i is nothing other than π_i. Dropping the irrelevant subscript i, we have

$$\pi = \frac{a^2}{9b}$$

If now there is exogenous shift of demand one simple representation is in terms of a shift in the parameter "a" in the demand function so that

$$\hat{\pi}/\pi = 2\frac{\hat{a}}{a} \text{ (assuming b constant)}$$

$$= 2g\text{, say.}$$

As an illustrative example to keep the algebra simple, assume that g is constant. Then

$$\pi = \pi(o).\ e^{2gt}. \quad \text{Choose } \pi(o) = 1.$$

Then the relationship between the profit margin π and rate of growth g is a simple exponential curve

$$\pi = e^{2gt}$$

And, of course, at $g = 0, \quad \pi = 1.$

In words, confronting a constant exponential growth of demand the duopolists plan on a profit margin that rises at twice that rate and the *μ-frontier* (on π - g plane) starts at $\pi = 1$ at $g = 0$ and rises exponentially (Fig. 28).

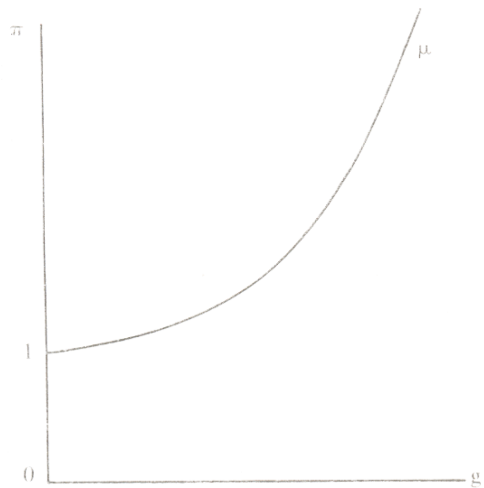


Fig. 28

VI.6. **A Non-Existence of Equilibrium** Theorem: This diagram should be interpreted with caution. It *looks* very similar to a curve relating profit (π) to time (t) for a constant growth rate (g), but it is not. The curve relates π to g which is a parameter. Hence, it has to be drawn for a fixed t, and answers the question, "what would be planned profit at a given t if g were to be a different constant?" For every g, as t rises, π rises. Hence, by varying t we get a family of curves, the lowest one standing for t = t_1, in Fig. 29.

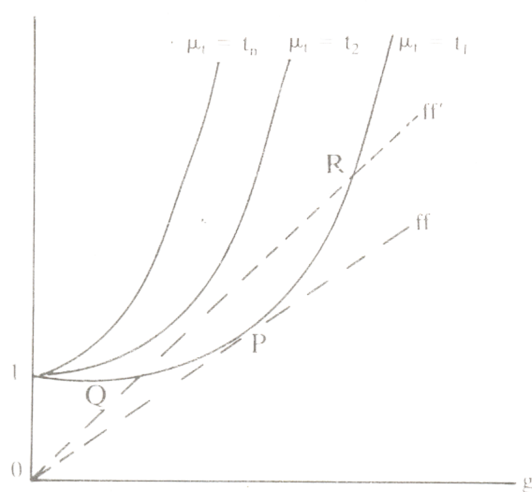


Fig. 29

VI.7. We now face problems. The trade-off involved in the J.L-functions so far in the analysis has completely disappeared; and it is easy to see that even the existence of a capitalist - (Wood -) equilibrium is not ensured if the finance frontier fails to intersect the curve μ drawn for $t = t_1$. We could be lucky and get a tangency solution as at P (Fig. 29) or, if ff' were higher placed (as in ff'), we would get two equilibria (up to a certain value of t) with the lower one being stable, the upper one being unstable. But all this is of academic interest unless we first resolve the basic question as to why would equilibrium fail to exist? The answer is that the strategic behaviour involved in the Cournot assumptions make the duopolists plan to grow *too fast.* The assumption of passive behaviour of the rival eliminates the trade-off between profit and growth rates so that whether the finance frontier is a straight line (as in Wood's model) or curvilinear (as in the ZIRE model) the existence of equilibrium is not guaranteed (Fig. 30).

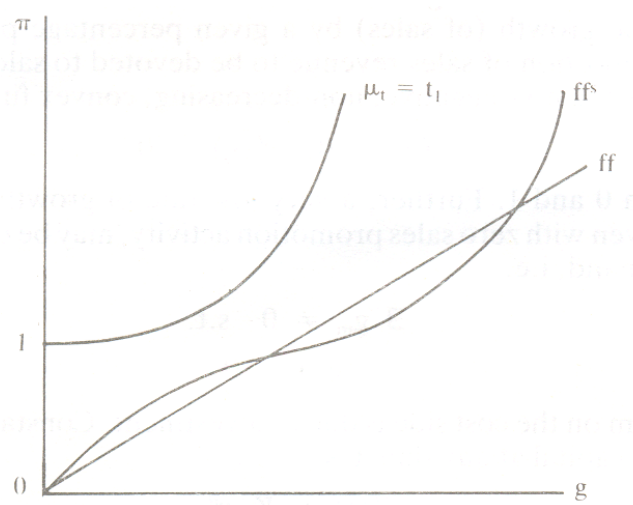


Fig. 30

In other words, the behaviour assumption involved in a Cournot duopoly are not compatible - on their own and without other new restrictions imposed elsewhere - with the kind of growth-profit trade-off required for the use of the μ-frontier as we need it.

VI.8. It is possible to interpret this negative result in two very different ways; one is to argue that this raises some doubts about the generality of our representation based as it is on Wood's model since it seems unable to accommodate an outstanding model of duopoly. The other is to argue that it highlights once more the weakness of the Cournot assumption especially when applied to even the simplest intertemporal model consisting of an exogenous growth of demand at a constant rate. We are inclined to take the second position and turn to a different model of oligopolistic behaviour where the tradeoff is clear cut but does admit of strategic behaviour. This is Solow's model, already referred to but we will give a slight reconstruction to illustrate our point though in Solow's analysis of value-maximizing firms strategic behaviour played no explicit role.

VI. 9. The Finance Constraint in Solow's Model of Oligopoly: We start with a brief outline of the model. An oligopolistic firm is choosing its rate of growth, g, and size of capital with which to start operations, K. Initial output Q is related to initial capital K by the relation.

$$Q = bK.$$

Labour and other running inputs are also combined in fixed proportions and all current costs are represented by a number of 'a' rupees per unit capital, or, a/b per unit of output. Capital cost is Rs. m per unit.

The crux of the matter is that to generate a growth of demand at the rate g the firm has to spend a fraction of gross revenue denoted by s(g) on sales promotion activities; obviously

$$0 \geq s(g) \leq 1.$$

It is further assumed that if g rises then s rises at an increasing rate, i.e., each increase of the growth (of sales) by a given percentage point requires an increasing proportion of sales revenue to be devoted to sales promotion. In other words, s is a non-negative, non-decreasing, convex function of g, i.e.,

$$s'(g) \geq 0, \; s''(g) \geq 0$$

lying between 0 and 1. Further, a very low rate of growth, say gm may be maintained even with zero sales promotion activity, may be due to exogenous growth in demand, i.e.

$$\exists \; g_m \neq 0 \quad \text{s.t.}$$
$$s(gm) = 0.$$

The last item on the cost side is due to investment. Constant growth at rate g implies that capital at any time t is

$$K(t) = K \; e^{gt}.$$

Hence, investment at each time t is given by

$$I \quad = \quad \hat{K}(t) \quad = \quad gKe^{gt}$$

and its cost is, obviously, Rs. $mgKe^{gt}$.

We are neglecting depreciation as it plays no role in the model, but can be easily included.

On demand, the assumption is of a constant elasticity schedule at the initial date given by

$$p \;=\; Q^{-1/n}$$

where p is product price and n is the constant price elasticity of demand. It would be possible to construct models where price p varies at each t; they would be algebraically messier; but we do not do it here to keep apart price competition and sales competition as sharply as possible.

It is now time to write down net profits at any t. Total revenue R at any t is given by

$$R(t) \quad = \quad pQ(t) \quad = \quad Q^{1/n}. \quad Q. \quad e^{gt}$$

$$= \quad Q^{1-\frac{1}{n}} \quad e^{gt} \quad = \quad Q^{\theta} \quad e^{gt}$$

$$= \quad (bK)^{\theta} \quad e^{gt} \quad \text{where} \quad \theta \quad = \quad 1-\frac{1}{n}$$

Total operating costs are

$$aKe^{gt} \; + \; mgk\, e^{gt}$$

$$= (a + mg)\; Ke^{gt}$$

Hence revenue net of costs is given by

$$b^{\theta}K^{\theta}e^{gt} - (a + mg)\; Ke^{gt}$$

So that profits are given by

$$\pi(t) \quad = \quad (1 - s(g))\,(b^{\theta}K^{\theta}e^{gt} - (a + mg)\,Ke^{gt})$$

$$= \; T(g) \;\; (b^{\theta}K^{\theta} \; - \; (a \; + mg) \;\; K) \;\; e^{gt}, \;\; T(g) \; = \; 1 - \; s(g)$$

With this choice of g, therefore, the present discounted value of the firm's net earnings, discounted at the market rate i, is given by

$$V(g) \quad = \quad \int_0^x T(g)\,(b^{\theta}K^{\theta} \quad - \quad (a + mg)\,K)\,e^{gt}, e^{-1t}\,dt$$

$$= \quad \frac{T(g)\,b^{\theta}K^{\theta} - (a + mg)\,K}{i - g}$$

For this to make sense, we need the restriction

$$i \; - \; g \; > \; 0 \;\; \text{to hold}$$

VI. 10. The function V(g) is the analogue of Wood's μ-frontier and hence we need to know how it behaves as g varies. By direct differentiation we get

$$\frac{\delta V}{\delta g} \quad = \quad \frac{(T + (i - g)T')\,b^{\theta}K^{\theta} - (a + im)\,K}{(i - g)^2}$$

where $$T' \quad = \quad \frac{dT}{dg}$$

Hence $$\frac{\delta V}{\delta g} \quad = \quad 0 \;\; \text{where}$$

and when $$K \quad = \quad \left[\frac{T + (i - g)\,T')\,b^{\theta}}{a + mi}\right]^n$$

Notice that if T + (i - g) T' is always negative. Then $\frac{\delta V}{\delta g}$ < 0 always, and we get the old μ-frontier (Fig. 31).

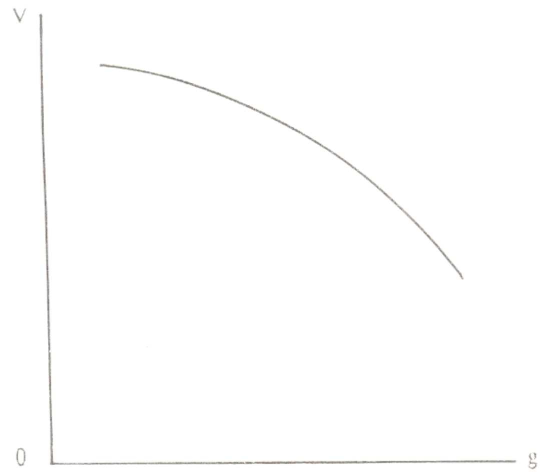


Fig. 31

On the other hand, since T'(g) = - s'(g) if we assume, consistently with the stipulation of s(g) made above, that T'(g) is small in size for small values of g and rises rapidly as g rises, then for any fixed K, δV/δg is positive for low g, comes to zero and turns negative. Thus we get the modified diagram (Fig. 32)

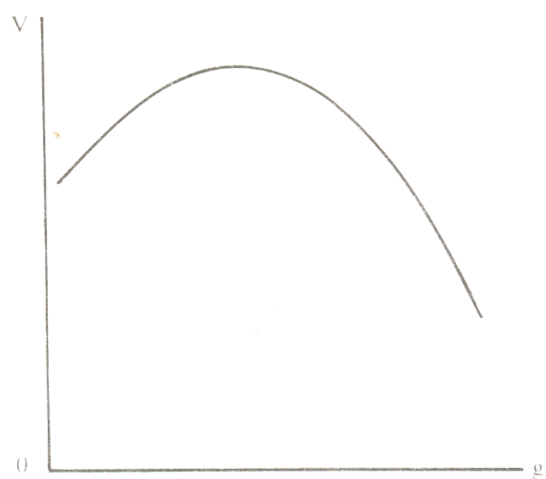


Fig. 32

In Solow's problem of value-maximization, one goes straightaway to the root K* of the equation $\frac{\delta V}{\delta g} = 0$, but not if we use the same structure to introduce strategic considerations in which case the initial size K can be chosen differently. Consider, for example a firm interested in *limit pricing* to keep rivals out. Suppose a price

$$P = p_L$$

is found, low enough, to hold off potential entrants. The firm works out the corresponding size K_L from the demand relation

$$Q = b^{\theta} K^{\theta} = (p_L)^{-n}$$

For all $K \geq K_L$, price will be lower than P_L and thus prevent entry. If this KL is less than K*, then this new consideration does not add anything to the model; the firm produces up to K* to sell at a price $p^* < P_L$. But should this size K_L, be greater than K* then the firm only produces the corresponding output Q_L (or more), selling at a price p_L, lower than or equal to p and achieving a value of V_L, (less than V*) so that for strategic reasons the solution V* is ruled out. See diagram 33:

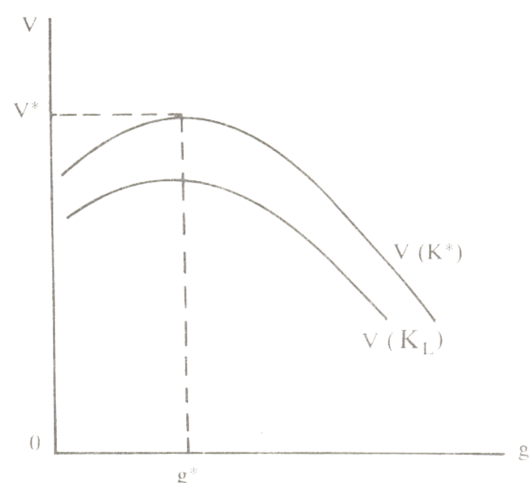


Fig. 33

There is one qualification of a technical nature that we must add to the interpretation of diagram 33. As drawn it implies that the V-curve for K_L lies below the one for K* suggesting that for fixed g, a higher K lowers V. It turns out that if a complete algebraic analysis is made then a lower K also would reduce V. We do not go into it here and the interested reader can find the details in Solow's paper. But the point here is that resort to limit pricing by an oligopolist in order to keep rivals out requires price to be *lowered,* so that output and capital stock have to be *higher*. Thus we can safely assume that the lower placed V-curves refer to a size of K greater than *K**.

What we are essentially arguing is that in Solow's model the nature of oligopolistic equilibrium would be seriously affected if the firms used *both* their control variables g and k strategically. It would not be impossible, for example, to reconstruct the duopoly model where both firms behave so and generate, for example, prisoners' dilemma type game situations.

VI.11. Reintroduction of the financial constraint will now further modify the outcome. For example, take the limit pricing model. The literature so far has investigated the consequences of chosing the policy V_L for example, and comparing the outcome with a competitive policy V_C, say, but not raised the question whether financial considerations could affect the outcome. They can. Consider the following cases familiar by now (Fig. 34).

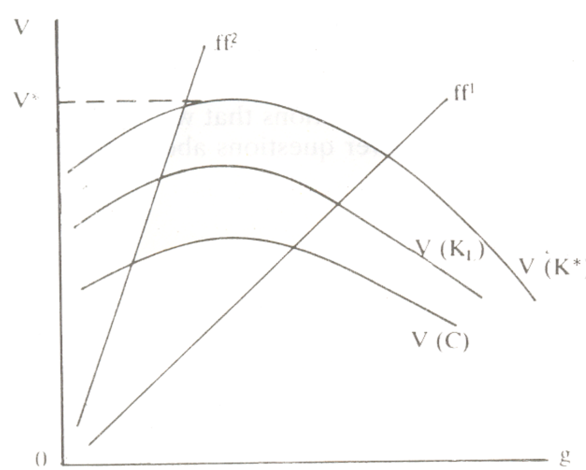


Fig. 34

The two straight line finance frontiers ff^1 and ff^2 stand for capitalist equilibria where financial constraints do or do not affect the strategy respectively. For, if ff^2 obtained, then V* is ruled out even without limit pricing! It is also clear that since the ZIRE finance frontier ff^s lies to the north-west of the corresponding finance frontier in a capitalist economy, in general there will be relatively less scope to pursue the limit price policy in ZIRE or less difference caused by it. We show a possibility in Fig. 35.

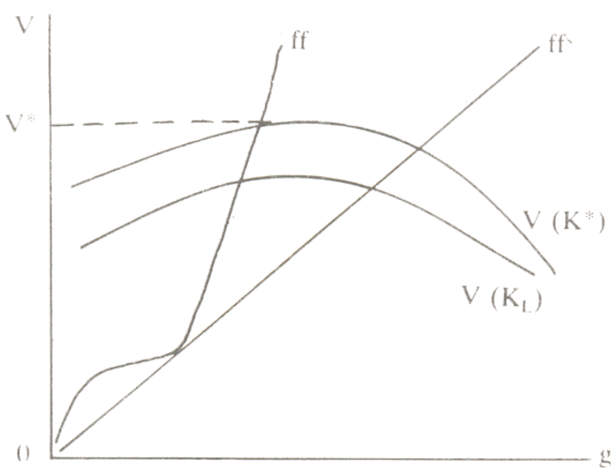


Fig. 35

Intuitively speaking, in the absence of loans, there is greater pressure in ZIRE on retained earnings and lesser scope to expand size and output to reduce price and prevent entry.

VI. 12. This, of course, is not a fully proved proposition but an indicative one. To rigorously prove it to be true or false, we need far stronger technical tools than have been used in this monograph. But we hope to have demonstrated the kind of considerations that will go into building up more comprehensive systems to answer questions about strategic behaviour and uncertainty.

VII. EXERCISES WITH AN EXHAUSTIBLE RESOURCE TYPE PRODUCTION MODEL

VII.1. We have constructed in some detail a simple representative model of a firm in a zero-interest-rate economy and contrasted the results in general terms with those of a capitalist firm. Often, however, the special features of a theory and the nature of the predictions that it makes are more sharply defined once it is applied to a special case which permits us to bring into use additional prior information and reasoning. With this object in view we shall try in this chapter to sketch a special case of some interest, that of a firm exploiting an exhaustible natural resource. The importance of the problem is obviously undeniable. To date, however, most of the work available has neglected the role played by financial constraints and analyzed the problem in the quantity-space. It is our hope that primitive as our analysis is, it might nevertheless help add something to the ongoing debate.

In what is to follow we shall try to make a distinction between those extractive industries in which current costs (including wages) are relatively high - as in coal - as compared to those in which initial investment is high but current costs are relatively low - as in oil. The reason is that while much of initial investment is equity financed, much of current costs, i.e., working capital. come out of short-run credit. Hence, conversion into a shares-economy will make different kinds of changes for the two situations.

VII.2. To fix our ideas and in the absence of any universally accepted framework let us try to first sketch a very simple model of an exhaustible resource that can be used to fit our formulation. We must warn, however, that no simple model can adequately handle the intertemporal choice problem which is so essential in formulating extraction policies. That policy affects the date at which the exhaustible resource reveals its specific attribute, i.e., it gets exhausted. The current profits from extraction have to be invested *elsewhere* so that after the resource gets exhausted those investments earn profits *elsewhere.* Varying that date one would get varying rates of extraction, current profits, investment and future profits. The decline in income in future due to the disappearance of the resource has to be compared with the increase in income due to that investment. It is not an easy problem to sort out and in our present, basically static, model we make no attempt to do so. I n any event, a model in which the flows per unit of time are the central parameters is incapable of adequately handling a stock-exhaustion problem.

VII.3. Even apart from the inter-temporal choice problem, the importance of the stock creates a fundamental difference in the nature of the analysis. When the equilibrium of the firm - or, for that matter, of an economy - is described, traditionally the *date* at which the description is made has been seen to be of no relevance whatsoever. Even in dynamic models that describe the behaviour of the economy over time, the steady state equilibrium, once attained, is independent of the calendar date. A problem in which the *size* of some stock plays a central role, however, intrinsically depends on the date at which the model is being applied because the stock size is an irreversible function of calendar time. The problem is in some sense the reverse of the famous learning by doing model in which the accumulated stock of knowledge affects productivity. In dynamic models such systems are described as “nonautonomous” meaning that the size of the endogeneous variables by themselves do not suffice to determine their laws of motion; the latter also depends on the calendar date. Time, denoted by t independently affects the nature of the dynamics.

VII.4. Obviously, it is impossible to analyze this problem in its entirety in a static model. What we shall try to do, therefore, is highlight and magnify those aspects of the problem that seem to derive from its peculiar dynamic structure. In so doing, we shall be looking for the so-called stylized facts of exhaustible resources as might be expected to stand out in the short-run. This will create one margin of imprecision in so far as these short-run stylized facts by themselves do not fully capture the true long-run properties of the exhaustible resource. That is a price we have to pay for the inadequacy of the static or comparative static structure of analysis in the present context. We are, as it were, using a proxy test for the real thing and other things - not truly qualified to belong to the class of exhaustible resources - might share the shortrun characteristics in some measure; slowly renewable natural resources are a prime example. As long as we keep this qualification in mind, no serious misapplication need occur. The first stylized fact seems to be that there is a *strong, persistent growth of demand independent of the firm's sales policy;* in effect one of the links between investment and profits are severed, the one which works through sales competition. The other major fact is that as the growth (of output) persists - and certainly if it increases - the *costs rise more than in proportion.* In other words, the economy of scale is offset by rising extraction cost. In practice this increase is embodied in different and more expensive techniques and technogies. At an aggregative level we cannot explicitly handle this shift; our formulation is a proxy.

VII.5. With this in mind we ask first, how would either a profit-maximizing firm or a firm out to maintain a given profit margin decide upon the target rate of growth? Let us take a very simple case of a downward falling demand relation given by

$$p = a - bx,$$

where p is price and x is quantity, and combine it with either of two costs relations:

(a) average cost = c, constant or

(b) average cost = c + dx

and work out the equilibria when demand grows exogenously, i.e., data is given.

VII.6. First, we take up the case of constant average costs c and investigate the equilibrium of the profit maximizing and the sales maximizing firm in turn.

(i) The *profit maximizing firm.* Here, a diagram illustrates the problem (Fig. 36).

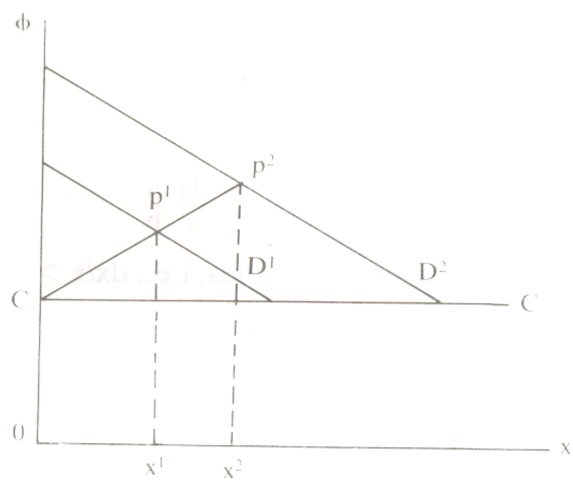


Fig. 36

D^1 and D^2 represent two positions of the demand curve, while cc is the con stant average cost. The locus of equilibrium points p^1, p^2 are the mid-points of the respective demand curves above the cc line (by the usual unit-elasticity rule). The change in x will be more than in proportion to the shift in demand curve because, in effect, for locating the equilibrium the relevant origin is c and not o. This is easily demonstrated algebraically.

$$\text{Profit per unit} = p - c = (a - c) - bx$$

Hence, total profit $\pi = (a - c)\, x - bx^2$, reaching maximum at

$$\frac{d\pi}{dx} = 0 = (a - c) - 2bx$$

giving equilibrium

$$x^o = \frac{a - c}{2b}$$

Hence for a proportional change da/a in demand we get

$$dx = \frac{da}{2b} \text{ and so}$$

$$\frac{dx}{x} = \frac{1}{x}\,\frac{da}{a}\,\frac{a}{2b}, \text{ for arbitrary x.}$$

But, $$x^o = \frac{a - c}{2b}$$

Hence $$\frac{dx}{x} = \frac{da}{a} \cdot \frac{a}{a - c} > \frac{da}{a}$$

(ii) Now let the firm operate with a target profit $\pi = \bar{\pi}$ per unit.

Then we have $$(a - c) - bx = \bar{\pi}$$

defined for $a - c > \bar{\pi}$ only.

Thus $$a\, dx = \frac{da}{b}$$

and $$\frac{dx}{x} = \frac{1}{x}\,\frac{da}{a}\,\frac{a}{b}$$

Hence our earlier conclusion again holds, i.e., $dx/x > da/a$.

Further, for getting the relationship between the profit margin $\bar{\pi}$ and the rate of growth dx/x rewrite the last equation as

$$\frac{dx}{x} = \frac{b}{(a - c) - \bar{\pi}} \cdot \frac{da}{a} \cdot \frac{a}{b} = \frac{da}{a} \cdot \frac{a}{(a - c) - \bar{\pi}}$$

As $\overline{\pi}$ rises, equilibrium x^o falls, firms achieve a higher profit by cutting back on output. The increase in output caused by an exogenous shift in demand therefore, causes a higher proportionate growth of output than would other wise be the case. As $\overline{\pi}$ rises, $\frac{dx}{x}$ *rises*. What is more relevant in the pre sent context is the implication of the reverse statement, which is that as the rate of growth of output $\frac{dx}{x}$, indicated earlier by the variable g, increases, the profit margin for the firm ($\overline{\pi}$) increases. **In** other words, the rising part of the frontier which we had earlier terminated by invoking declining demand, diseconomies of scale and increasing sales competition from rivals, now becomes a persistent tendency.

But this is the model of the capitalist firm for which the finance-frontier is also an upward rising line. Hence, as we can suspect, the firm might be in an unstable equilibrium where the two frontiers intersect (Fig. 37).

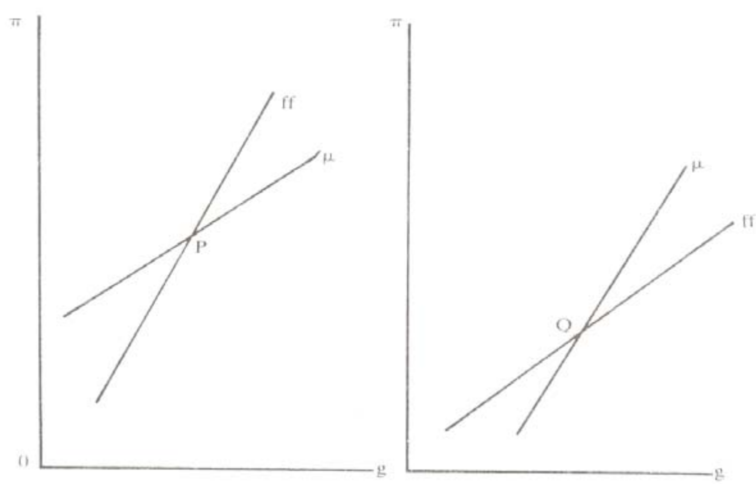


Fig. 37a Fig. 37b

In Fig. 37 the cross-hatched area is the feasible set. **In** panel (a) it describes a stable equilibrium at P but in panel (b) the equilibrium at Q is unstable. *Relatively* speaking the conditions of finance are much less stringent in (b) than in (a) - the need to plough back additional profit into investment increases more slowly in (b) as compared to (a). Faced with a strong enough exogenous growth of demand, the firm might keep on increasing profit-margins and extraction rates simultaneously and indefinitely.

Without the use of the finance frontier, this essential instability problem cannot be demonstrated.

VII.7. Let us now turn to the rising-cost case. As before we stay with simple linear relations for the ease of exposition. To this end, we retain the older demand relation

$$p = a - bx$$

and use, for average cost, the relation

$$p = c + dx, \quad c, d \text{ constants } > 0$$

and carry out the analogous exercise.

(i) For the profit-maximizing firm the condition for equilibrium of equality between marginal revenue and marginal cost here implies

$$a - 2bx = c + 2dx$$

or
$$x = \frac{a-c}{2(b+d)}$$

From this we easily get, for proportional changes in x. a and c, the equation

$$\frac{dx}{x} = \frac{1}{x}\frac{d(a-c)}{2(b+d)}$$

$$= \frac{1}{x}\left| a.\frac{da}{a} - c\frac{dc}{c} \right| \frac{1}{2(b+d)}$$

Now writing g for the rate of growth of any variable we get

$$g_x = \frac{1}{x}\left[ag_a - cg_c\right]\frac{1}{2(b+d)}$$

$$= \frac{ag_a - cg_c}{a-c}. \quad (\text{since } x = \frac{a-c}{2(b+d)}$$

$$= \frac{d(a-c)}{a-c}$$

There is no unequivocal relation between the rate of growth of equilibrium output g. and that of demand, g_a. any more without specifying how costs behave. The term in brackets in the right-hand side of the equation for g_x reflects the change in the gap between a and c caused by exogenous increases in demand and cost respectively. If at the margin costs rise faster than demand (i.e, $g_c > g_a$) then g_x will be falling; for

$$ag_a < cg_c$$

g_x might actually turn negative.

Since $d_x = \frac{1}{2(b+d)}(da - dc)$, dx has the same sign as (da - dc). Hence we get Fig. 38.

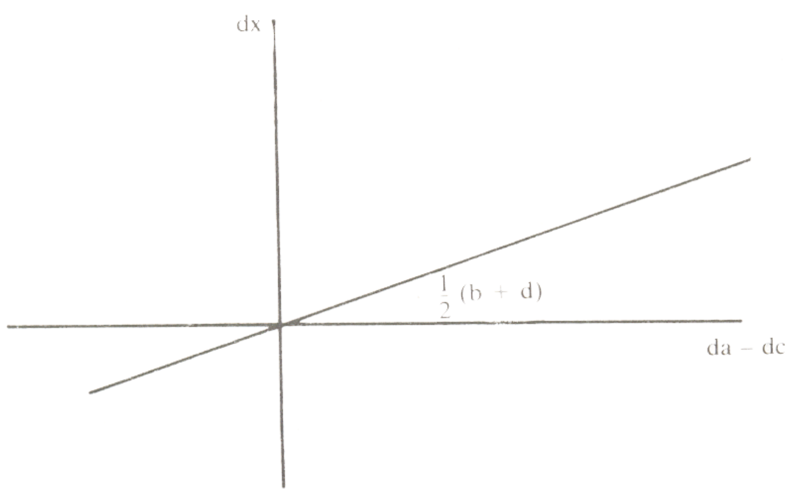


Fig. 38

(ii) Identical results hold for the sales-maximizing firm. Since profit per unit is

$$a - bx - (c + dx)$$

we have $$\bar{\pi} = (a - c) - x(b + d)$$

or $$x = \frac{(a - c) - \bar{\pi}}{b + d}$$

Carrying out similar manipulations, we get

$$g_x = \frac{dx}{x} = \frac{1}{x}[ag_a - cg_c]\frac{1}{(b+d)} = \frac{ag_a - cg_c}{(a-c)-\overline{\pi}}$$

Apart from a a doubling of the slope, Fig. 38 applies. However, the relationship between $\overline{\pi}$ and g_x is more complicated. Cet par, a rise in $\overline{\pi}$ raises g_x for reasons identical with the constant cost case. But if as 'a' rises, pulling up x, c rises faster (so that the 'cet par' assumption is violated), g_x might fall. In words, the case boils down to this: if as demand rises (da > 0) a higher profit margin is sought ($\overline{\pi}$ rises) then the rate of growth of output (dx) might turn negative, depending upon how fast costs rise (i.e., the size of dc). A sufficiently sharp increase in costs might make a reduction of output the optimal policy for firms trying to protect a given margin of profit.

If g_c is zero so that cost at the margin increases but there are no exogenous increases in the cost *schedule* then

$$\frac{dx}{x} = \frac{1}{x}\cdot\frac{ag_a}{b+d} = \frac{ag_a}{(a-c)-\overline{\pi}}$$

and our earlier conclusions hold. A higher $\overline{\pi}$ implies a higher g_x.

If the gap between demand and cost schedules stay unchanged i.e., d (a - c) = 0 then $g_x = 0$.

Thus, for the major alternatives, we get two situations. If ($ag_a - cg_e > 0$ we have $\overline{\pi}$ and g_x positively related as before. The equilibrium is stable or unstable depending on whether the μ-frontier or the ff-schedule is steeper. For ($ag_a - cg_e < 0$, an increase in $\overline{\pi}$ makes g_x a larger negative number (Fig. 39).

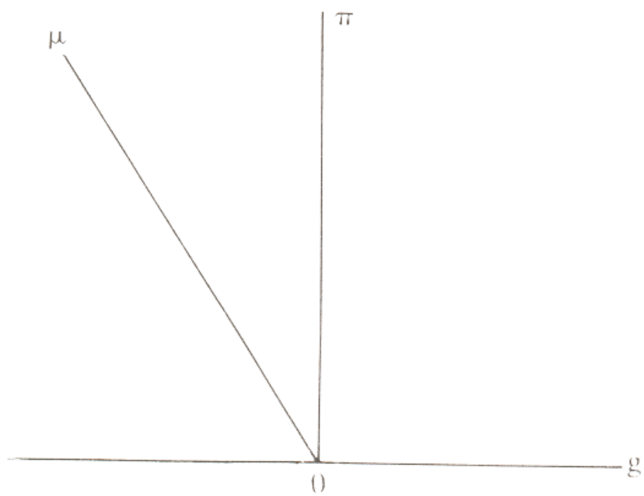


Fig. 39

The important point is that as long as ff is in the first quadrant, the equilibrium of a capitalist firm will also always occur in it. It may be stable or unstable but both g and $\overline{\pi}$ will be positive.

VII.8. We can now begin to modify the finance frontier as previously done and very quickly trace the major qualitative differences that may occur. As will be recalled, the ff line is changed into an upward rising curve with a non-convex region in it. Since in the present situation π also rises, we again get into the quantitative problem of deciding which rises faster, something on which no prior information is available. On the other hand the same degree of freedom leaves open scope for active policy intervention which might choose socially preferred solutions through simple variations of financial parameters. To illustrate, take the case where ff^s rises quite rapidly. Recall that it is a product of the form.

$$(\alpha - \beta (g))\, g$$

where $(\alpha - \beta (g))$ eventually approaches a constant, ex, since B(g) asymptotically approaches zero. (But that happens only for large values of g, so large that we might disregard them as being unrealisable in practice). Thus the product can be expected to increase quite rapidly after the non-convex zone is traversed. In that case the outcome looks like Fig. 40.

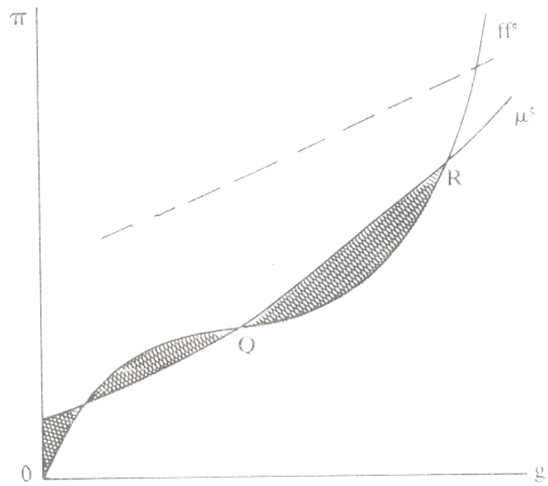


Fig. 40

There are three equilibria, P, Q and R and the feasible sets are cross-hatched. The unstable one, i.e., Q, comes in the middle. Hence the system will never be globally unstable. Further, exogenous shifts in demand that shift μ^s upward will tend to lean towards generating a single, stable equilibrium. Unless demand *at the margin* increases very fast inducing an anticlockwise swing of μ^s, the instability problem will never be as alarming in the present case as it was in the case of the capitalist firm. In fact for all reasonable parameter values the equilibrium will be a stable one.

VII.9. But let us now turn to the other case depicted by diagram 39 of a negative rate of growth and ask, could this be relevant in the context of ZIRE. This, in simple terms, is the case of conservation and was seen to be unachievable for a capitalist equilibrium. The basic reason was, of course, the fact that for such a firm the finance frontier was always in the first quadrant so that a μ-locus in the second could never yield an equilibrium. The reason why we rejected the possibility of the finance frontier being in the second quadrant is inherent in the logic of how it is set up. To repeat, retained earnings had to meet the gap between liquid reserves and external finance and the latter - a loan - was taken to be a fixed contractual cost proportional to investment. For a firm in ZIRE, however, external finance is not a contractual cost but related to its profit - it is its dividend payment obligation. Suppose, for example, a firm like this confronts a rising cost situation that sends the μ-curve into the second quadrant. A capitalist firm in such a situation can at most send interest cost to zero by sending investment to zero i.e., from the equation

$$R = rP \geq (1 + f)\ I - xI$$

the maximum reduction in financial liability occurs at zero investment. It still does not get the conservationist equilibrium in the second quadrant.

But while a firm cannot pay interest to itself, it can pay dividends to itself by reducing its capital base. A contracting capitalist firm will find it very hard to raise a loan. For such a firm, we have to add the additional constraint.

$$x = 0 \quad \text{for } I \leq 0.$$

A ZIRE firm does not, or need not obey such a restriction. While a firm buying up its own shares in order to reduce its dividends payments obligation seems to be either fanciful or questionable business practice in a capitalist market economy, we must keep in mind the prospect that once the credit market disappears, a firm will definitely try to reduce its dividend payment obligations when confronted with zero or negative growth of revenue. In other words, it will try to *disinvest.* Take for example the case where, in ZIRE,

$$x^s < (1 + f) \quad \text{and } I < 0.$$

The finance frontier now extends into the second quadrant *after passing through the origin.* It must, therefore, cut the μ-curve. (which is now a straight line through the origin) several times, including at the origin, depending, once again on the actual shape of $(1 + f - x^s(g))$. We show a possible case below (Fig. 41).

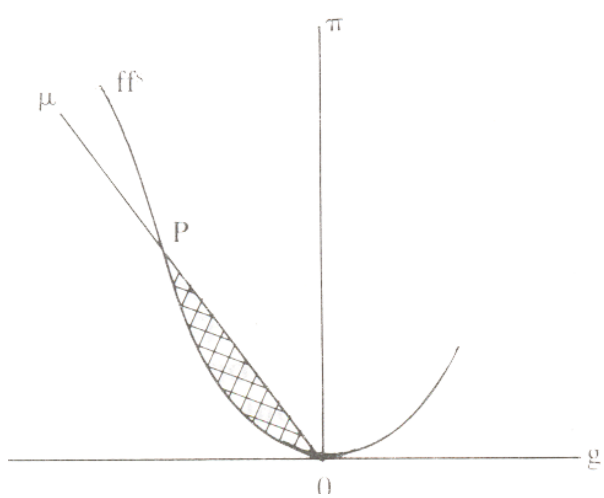


Fig. 41

Since the feasible set is the area below, and above ff^s, the situation depicted at P is a stable, conservationist equilibrium.

VII.10. In purely qualitative terms, not much can be claimed about the certainty of achieving a solution as in fig. 32, but the general result is clear. The discussion of extraction policies is incomplete without a reference to the financial side of the model. Once that is brought in, the capitalist firm is more likely to be in unstable equilibria than the **ZIRE** firm; and the capitalist firm is unlikely, on its own, to follow conservationist policies that are open to the **ZIRE** firm. These broad qualitative results are just about as far as we can go with our simple model but they are worth more detailed exploration.

On the other hand if these results seem to be a bit unusual, it is instructive to ask what the model would look like if the finance constraint is neglected completely. Then the distinction between the capitalist and the shares economy disappears and both are left with only the μ-frontier (or its equivalent, depending upon how we model it) and hence a degree of freedom in the system. One would then have to introduce an additional restriction somewhere, and in a dynamic model it would most probably be built around the decline of the stock as the system moves over time. Typically, one would construct a demand schedule and compute g_x (of sections A.6 - A. 7 above) based on demand considerations including the actions of rival firms and try to capture in g_c the impact of stock decline. A priori it is not very clear that the neglect of the finance-constraint is innocuous except under very strong conditions on the nature of competition, demand and other parameters of the system.

VIII. CONCLUSION

It is not very easy to summarize our results and present them in sharp contrast to traditional ones, for what we have really sought to describe is an alternative methodology rather than an alternative set of results. The lack of a concrete, factual point of reference, in our judgement, makes the entire analysis - and any analysis of its kind - necessarily hypothetical and to that extent enhances the need to emphasize important distinguishing features of the system under study, rather than suhtler, albeit intellectually more exciting, points of detail.

From this point of view, undoubtedly the single most prominent item in this monograph has been the use of the financial constraint in describing the equilibrium of the firm. We started with the original work of Wood and used it to demonstrate interior solutions even in the presence of scale economies (Ch. III). Next, we modified the finance frontier for a zero-interest-rate economy where loan financing is absent, and made up for by equity investment. With the reconstructed finance frontier we then investigated the various types of equilibrium that can emerge (Ch. IV). Under reasonable assumptions, the new economy seems to generate equilibria that involve higher growth rates and lower profit margins than the other one did. We investigated how scale economies and technical progress are handled in the new set-up and then passed on to questions about the nature and role of competition (Ch. V) and the differences caused by strategic behaviour in Ch. VI. (The last one is rather incomplete because geometry is not an adequate tool any more for this class of questions). In conclusion, we gave an illustration of the use of our model by an analysis of exhaustible resources in the Ch. VII. While little empirical evidence can be provided to justify the specific forms of important functions used, it nevertheless helps bring out some important messages.

First and foremost is the point that even though the *actual shapes* might be different, the combined μ-ff diagram perhaps provides the easiest and best format in which to cast the problem; even *for* the model of a capitalist *firm,* it is perhaps the only one which allows financial considerations to playa direct role in the determination of equilibrium. In this sense, it is the logical counterpart of Keynes's analysis of macroeconomic equilibria, where money plays a central role, to the micro-economics of investment in firms. This is an important point because right now in spite of hectric research and interest in Keynesian general equilibrium theory, the corresponding re-examination of micro-economics has not happened.

Secondly, the specific finance frontier we used introduces a sharp non-convexity in the feasible set and leads up to multiple equilibria. This is dearly more general than Wood's model but that is not all. The non-convexity in question is due *not to technological but to institutional* forces and thus cannot be gotten rid of by known assumptions like diminishing returns or quasi-concave preferences. One should realize that the assumption of a perfect capital market makes the set of intertemporal consumption possibilities convex and that is indeed a very useful prop in Fisherian analysis. Over here that is precisely what has to be dropped and the consequences as in our model, may be drastic.

Thirdly, about the special cases dealt with we can only claim that they were chosen with a preconceived idea that they are important by any standards and hence at least some effort needs to be made to see what they look like in the present context. About the exhaustible resources model the hypothesis being suggested is that the operation of the financial constraints in the new system generally favours a conservationist policy. In the strategic actions model, it is that the new system is likely to deter the use of entry-preventing price strategies. Even though not worked out formally these hypotheses are worth checking out in detail. The second one, in particular, once again underscores the need to pay closer attention to the role of the money market in oligopolistic equilibrium. If access to loanable funds provides greater scope for limit-pricing strategies then contrary to conventional wisdom a ‘free’ capital market seems to create a restricted product market. The least we can claim is that we have here, clearly, areas that merit further research.

The Zero-interest-rate-economy, as and when it starts to function, will provide the real refutations of what has been said here, but about one thing there ought to be no doubt. There are unlikely to be any easy answers or quick solutions. The claim of either ‘obvious’ improvement or ‘obvious’ inefficiency is far too naive.

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